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COMBINED TURBINE-MAGNETOHYDRODYNAMIC BRAYTON CYCLE POWER SYSTEM FOR SPACE AND GROUND USE

by Lester D. Nichols

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Cleveland, Ohio 44135

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16. Abstract

A combined turbine-MHD generator operating in a Brayton cycle with a NERVA nuclear reactor is considered, both for use in space and on the ground. The combined system is compared with an all-MHD Brayton system and an all-turbine system. The combined cycle systems have higher thermodynamic efficiencies than the other systems. The combined system with 1500 K turbine inlet and the all-MHD system with generator efficiency of 0.8 have the lowest specific recuperator plus radiator mass of those considered. But the combined system considered has an average radiator temperature of 200 to 250 K lower than the other. For ground use, a cycle efficiency of greater than 0.55 can be achieved.

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COMBINED TURBINE-MAGNETOHYDRODYNAMIC BRAYTON CYCLE POWER SYSTEM FOR SPACE AND GROUND USE

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SUMMARY

A combined turbine-magnetohydrodynamic (MHD) generator operating in a Brayton cycle with a NERVA nuclear reactor is considered, both for use in space and on the ground. The combined system is compared with an all MHD Brayton system and an all-turbine system. Generator efficiencies of 0.8 and 0.55 and turbine inlet temperatures of 1500 and 1250 K were considered. Reactor exit temperature of 2500 K was considered for the combined system and the all-MHD system. Reactor exit temperatures of 1500 and 1250 K were considered for the all turbine system. For space application, the specific recuperator plus radiator mass was minimized. The overall cycle pressure ratio, the pressure loss due to friction in the components, the overall cycle thermodynamic efficiency, and the minimum specific radiator area were determined for four combined cycles, two all-MHD, and two all-turbine systems. The combined cycle systems have higher thermodynamic efficiencies than the other systems. The all-MHD system with a generator efficiency of 0.8 and the combined system with a generator efficiency of 0.8 and turbine inlet temperature of 1500 K have the lowest specific mass. But the combined system has a radiator temperature 200 to 250 K lower. For ground use, the cycle efficiency will be greater than 55 percent if the generator efficiency is 0.8, and greater than 40 percent if the generator efficiency is greater than 0.55.

INTRODUCTION

At present nuclear Brayton cycle systems are being considered for generating power in space (as well as on the ground) at maximum temperatures less than 2000 K. These systems would have lower specific radiator area if the working fluid maximum temperature were increased. The maximum temperature is presently set by reactor and turbine limitations. The NERVA nuclear reactor technology has the potential of providing an inert gas working fluid at a higher temperature (up to 2500 K). An MHD generator has the potential to utilize efficiently this high temperature working fluid, whereas a gas turbine would likely require cooling. A system with the combination of MHD generator and NERVA reactor has been studied by Holman and Way (ref. 1) for generating power in space in a conventional Brayton cycle. Their results are encouraging.

Replacing the turbine by an MHD generator makes no change in the thermodynamic analysis, but the power to drive the compressor must still come from the expander - whether it is a turbine or an MHD generator. In the conventional Brayton cycle the turbine power is transferred mechanically (usually on the same shaft) to the compressor. This transfer is nearly 100 percent efficient. If the expander is an MHD generator, then some other means must provide the power to drive a rotating compressor. Keeping a turbine in the high-temperature system to drive the compressor would have advantages over an electric motor drive.

Since an MHD generator is efficient at the high temperatures (2500 K) whereas an uncooled turbine is efficient at the lower temperatures (1500 K), each expander will be used at the temperature where its performance is best. This division of expander work takes advantage of the high-temperature capability of both the NERVA reactor and MHD generator and at the same time avoids using the MHD generator at lower working-fluid temperatures where it becomes less efficient. Also, an uncooled gas turbine will be used at the maximum possible temperature at which it can operate efficiently, thus providing compressor power with maximum transfer efficiency and minimum weight. This combination has been suggested previously (refs. 2 to 5) for ground use. These authors conclude that this combined Brayton cycle system could provide a very attractive power-generation system for use on Earth at temperatures that are possible with the NERVA reactor. We propose to study the possibility of using this system in space.

However, two changes will be made making this system different from the conventional Brayton cycle considered for space use. First, a turbine with reheat will be employed. This procedure is used in Rankine cycles in order to maintain a high quality working fluid, but is not usually employed in Brayton cycles, probably because of the pressure losses associated with the heat transfer. However, in this application, a very large temperature difference is available for transferring the heat, and as a result, the heat transfer can probably be done with little pressure loss. Second, compressor intercooling will be used. (This is already assumed for the cycles to be used on the ground.) In space, the intercooling will provide a more nearly isothermal radiator, which in turn will provide a smaller radiator area for the cycle.

Comparisons of the combined turbine-MHD system for use in space will be made on the basis of the required specific radiator and recuperator masses and the corresponding overall cycle efficiency. The combined cycles will be compared to both an all-MHD and all-TURBINE conventional Brayton cycle with compressor intercooling.

CYCLE ANALYSIS

Overall Thermodynamic Cycle Efficiency

The combined turbo-MHD Brayton cycle schematic diagram is shown in figure 1. The intercooled Brayton cycle is formed by removing the turbines (i.e., state points 3 to 7) and the high temperature (state point 16) recuperator. The overall thermodynamic cycle efficiencies of both cycles are calculated in terms of the state points by expressing the net electrical power generated and the net thermal power delivered by the reactor in terms of the state points, and then calculating the efficiency as the ratio of the two. The

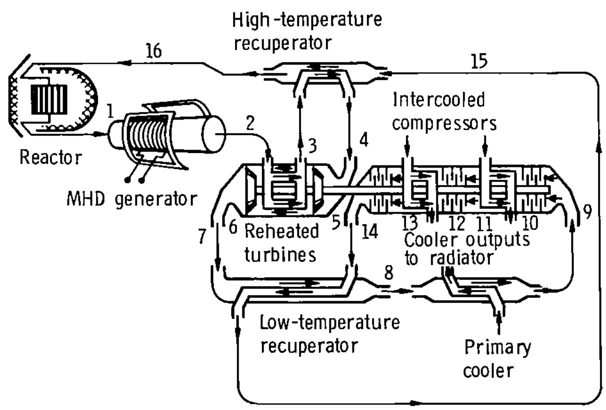
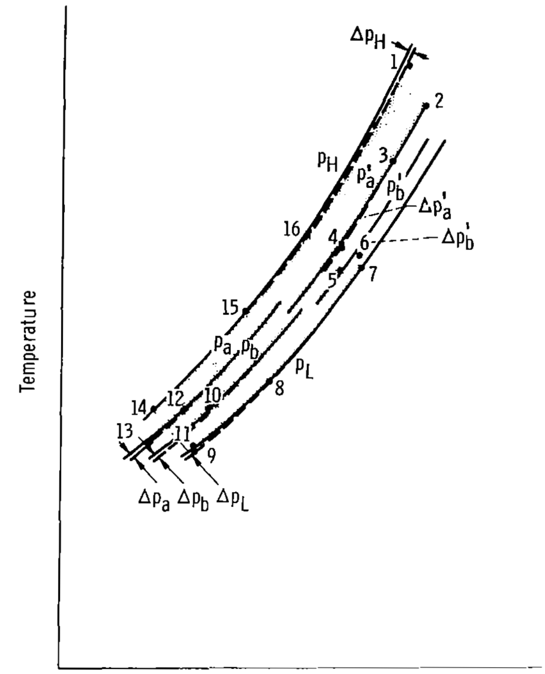


Figure 1. - Turbo-MHD power system.

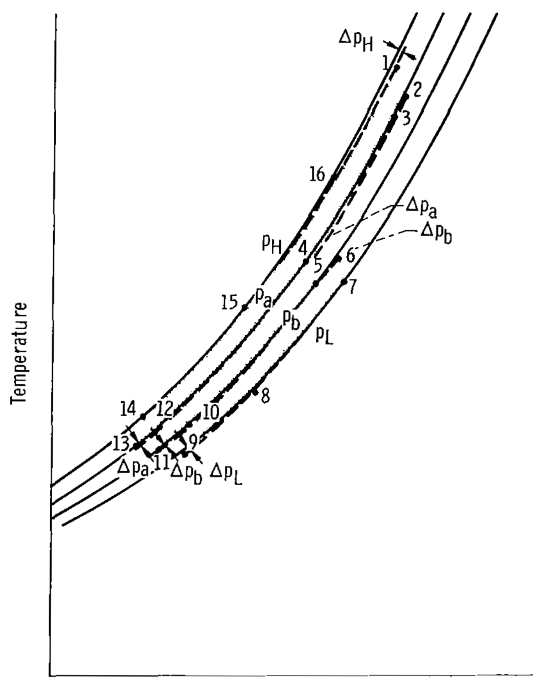
working fluid is assumed to be a perfect gas with constant specific heat. The generator and turbine efficiencies are defined as the actual change in temperature divided by the isentropic change between the same pressure limits. A superconducting magnet is assumed. The required refrigerator power and radiator weight is neglected in this analysis. All symbols are defined in appendix A.

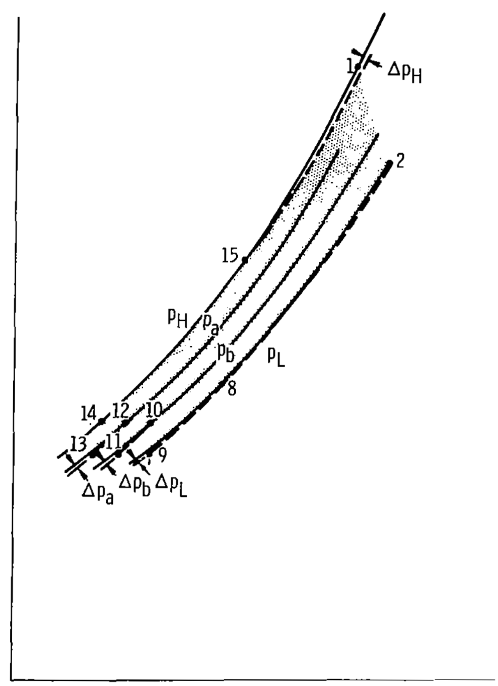
Combined cycle. - Consider the temperature-entropy diagram shown in figure 2(a). The generator power is:

$$P_{gen} = wc_p(T_1 - T_2) \tag{1}$$



(a) Combined for ground-based use.





Entropy

(b) Combined for use in space.

(c) Intercooled Brayton for space use.

Figure 2. - Brayton cycle temperature-entropy diagram.

The exit temperature can be expressed in terms of the inlet temperature, the generator efficiency, and the pressure ratio. Then the generated power can be written as

$$P_{gen} = wc_pT_1\eta_{gen}\left[1 - \left(\frac{p_a'}{p_H - \Delta p_H}\right)^{(\gamma-1)/\gamma}\right] \tag{2}$$

The power output from the two stages of the turbine is

$$P_{turb} = wc_p(T_4 - T_5 + T_6 - T_7) \tag{3}$$

Again, the exit temperature of each stage can be expressed in terms of the inlet temperature, the turbine efficiency (assumed to be the same for both stages), and the turbine pressure ratio. Then the turbine power can be written as

$$P_{turb} = wc_p\eta_{turb}\left\{T_4\left[1 - \left(\frac{p_b'}{p_a' - \Delta p_a'}\right)^{(\gamma-1)/\gamma}\right] + T_6\left[1 - \left(\frac{p_L}{p_b' - \Delta p_b'}\right)^{(\gamma-1)/\gamma}\right]\right\} \tag{4}$$

The power delivered by the turbine will be assumed equal to the power required by the compressor. It will be assumed that the two stages of the turbine have the same inlet temperature and the same outlet temperature; that is

$$T_4 = T_6 \tag{5}$$

and

$$\frac{p_b'}{p_a' - \Delta p_a'} = \frac{p_L}{p_b' - \Delta p_b'} \tag{6}$$

Then the power output from the two stages of the turbine becomes

$$P_{turb} = wc_p\eta_{turb}2T_4\left[1 - \left(\frac{p_L}{p_b' - \Delta p_b'}\right)^{(\gamma-1)/\gamma}\right] \tag{7}$$

The power required by the three compressor stages is

$$P_{comp} = wc_p(T_{14} - T_{13} + T_{12} - T_{11} + T_{10} - T_9) \tag{8}$$

The compressor stage exit temperatures can be written in terms of the compressor inlet temperature, the (equal) compressor efficiencies, and the pressure ratios, so that the compressor work can be written as

$$P_{comp} = \frac{wc_p}{\eta_{comp}}\left\{\left[\left(\frac{p_b}{p_L - \Delta p_L}\right)^{(\gamma-1)/\gamma} - 1\right]T_9 + T_{11}\left[\left(\frac{p_a}{p_b - \Delta p_b}\right)^{(\gamma-1)/\gamma} - 1\right] + T_{13}\left[\left(\frac{p_H}{p_a - \Delta p_a}\right)^{(\gamma-1)/\gamma} - 1\right]\right\} \tag{9}$$

Assuming that the three stages of the compressor have equal inlet temperatures and outlet temperatures; that is,

$$T_9 = T_{11} = T_{13} \tag{10}$$

and

$$\frac{p_b}{p_L - \Delta p_L} = \frac{p_a}{p_b - \Delta p_b} = \frac{p_H}{p_a - \Delta p_a} \tag{11}$$

the work required by three stages of compression becomes:

$$P_{comp} = \frac{3wc_pT_{13}}{\eta_{comp}}\left[\left(\frac{p_H}{p_a - \Delta p_a}\right)^{(\gamma-1)/\gamma} - 1\right] \tag{12}$$

There is a pressure loss associated with each heat-transfer process. We will assume that the fractional pressure drop for each process is the same; that is

$$\frac{\Delta p_a'}{p_a'} = \frac{\Delta p_b'}{p_b'} = \frac{\Delta p_H}{p_H} = \frac{\Delta p_a}{p_a} = \frac{\Delta p_b}{p_b} = \frac{\Delta p_L}{p_L} = \frac{\Delta p}{p} \tag{13}$$

Introduce the parameter

$$\delta \equiv \left(1 - \frac{\Delta p}{p}\right)^{(\gamma-1)/\gamma} \tag{14}$$

and define the isentropic temperature ratios corresponding to the pressure ratios

$$y_{gen} \equiv \left(\frac{p_a'}{p_H}\right)^{(\gamma-1)/\gamma} \tag{15a}$$

$$y_{turb} \equiv \left(\frac{p_b'}{p_a'}\right)^{(\gamma-1)/\gamma} = \left(\frac{p_L}{p_b'}\right)^{(\gamma-1)/\gamma} \tag{15b}$$

$$y_{comp} \equiv \left(\frac{p_a}{p_H}\right)^{(\gamma-1)/\gamma} = \left(\frac{p_b}{p_a}\right)^{(\gamma-1)/\gamma} = \left(\frac{p_L}{p_b}\right)^{(\gamma-1)/\gamma} \tag{15c}$$

$$y \equiv \left(\frac{p_L}{p_H}\right)^{(\gamma-1)/\gamma} \tag{15d}$$

As a result of these definitions, it is obvious that

$$y_{gen} y_{turb}^2 = y_{comp}^3 = y \tag{16}$$

Now, the generator power (eq. (2)) can be written as

$$P_{gen} = wc_p T_1 \eta_{gen} \left(1 - \frac{y}{\delta y_{turb}^2}\right) \tag{17}$$

the turbine power (eq. (7)) can be written as

$$P_{turb} = 2wc_p T_4 \eta_{turb} \left(1 - \frac{y_{turb}}{\delta}\right) \tag{18}$$

and the compressor power from equation (12) can be written as

$$P_{comp} = 3wc_p \frac{T_{13}}{\eta_{comp}} \left(\frac{1}{y^{1/3}\delta} - 1\right) \tag{19}$$

It is useful to define parameters which are proportional to the work of the components

$$G \equiv \eta_{gen}\left(1 - \frac{y}{\delta y_{turb}^{2}}\right) \tag{20a}$$

$$B \equiv \eta_{turb}\left(1 - \frac{y_{turb}}{\delta}\right) \tag{20b}$$

$$C \equiv \frac{1}{\eta_{comp}}\left(\frac{1}{y^{1/3}\delta} - 1\right) \tag{20c}$$

Then the exit temperatures from these components can be expressed in terms of these parameters as

$$T_2 = T_1(1 - G) \tag{21a}$$

$$T_5 = T_7 = T_4(1 - B) \tag{21b}$$

$$T_{10} = T_{12} = T_{14} = T_{13}(1 + C) \tag{21c}$$

The overall thermodynamic cycle efficiency for the combined cycle can now be determined. The thermal power from the reactor to the working fluid is

$$Q = wc_p(T_1 - T_{16}) \tag{22}$$

Then the overall thermodynamic cycle efficiency can be written as

$$\eta = \frac{P_{gen}}{Q} = \frac{T_1 - T_2}{T_1 - T_{16}} \tag{23}$$

The temperature of the working fluid entering the reactor can be expressed in terms of the other cycle temperatures and component parameters. The heat exchangers will be described in terms of parameters defining their effectiveness. In the case of the low-temperature recuperator, the heat picked up by the high pressure gas will be assumed equal to the heat lost by the low pressure gas. This heat will be expressed in terms of the maximum amount of heat transferrable

$$\eta_{rec} \equiv \frac{T_7 - T_8}{T_7 - T_{14}} = \frac{T_{15} - T_{14}}{T_7 - T_{14}} \tag{24}$$

In the case of the high-temperature recuperator and the reheat heat exchanger, the amount of heat that is lost by the lower pressure gas (from generator outlet to turbine inlet) will be specified to provide the required reheat and provide the required turbine-inlet temperature. This amount of heat must be transferred, regardless of the effectiveness of the heat exchangers. In order to assure that this occurs, the high-temperature heat exchangers will be designed so that only a fraction of the heat that is transferred from the gas being cooled is picked up by the gas being heated. This fraction, called a loss parameter L_h will be introduced. It will be the same for both high-temperature heat exchangers. The loss parameter is defined as

$$L_h \equiv \frac{T_6 - T_5}{T_2 - T_3} = \frac{T_{16} - T_{15}}{T_3 - T_4} \tag{25}$$

It is now possible to express the reactor inlet temperature in terms of component parameters and other cycle temperatures. The reactor inlet temperature can be written from equation (25) as

$$T_{16} = T_{15} + L_h(T_3 - T_4) \tag{26}$$

Also, using equation (25) along with equations (5) and (21b), one has

$$T_3 = T_2 - \frac{T_6 - T_5}{L_h} = T_2 - B\frac{T_4}{L_h} \tag{27}$$

Then, equation (26) can be rewritten using equation (27):

$$T_{16} = T_{15} + L_h(T_2 - T_4) - BT_4 \tag{28}$$

But, from equation (24):

$$T_{15} = \eta_{rec}T_7 + (1 - \eta_{rec})T_{14} \tag{29}$$

so that equation (28) can be written (using eq. (29)):

$$T_{16} = T_{14}(1 - \eta_{rec}) + \eta_{rec}T_7 + L_h(T_2 - T_4) - BT_4 \tag{30}$$

Finally, substitute for T_2, T_7 and T_{14} from equation (21) so that the reactor inlet temperature becomes

$$T_{16} = T_{13}(1 - \eta_{rec})(1 + C) + \eta_{rec}T_4(1 - B) + L_hT_1(1 - G) - T_4(L_h + B) \tag{31}$$

The efficiency can be determined from equation (23) by substituting for T_2 from equation (21a) and for T_{16} from equation (31):

$$\eta = \frac{G}{(1 - L_h) + L_hG + \frac{T_4}{T_1}\left[B(1 + \eta_{rec}) - \eta_{rec} + L_h\right] - \frac{T_{13}}{T_1}(1 + C)(1 - \eta_{rec})} \tag{32}$$

There are still two temperature ratios remaining to be specified in order to calculate the efficiency in terms of the overall cycle pressure ratio, loss parameters, and component parameters. One of these, the ratio of turbine inlet to generator inlet temperatures, will be specified directly. The other ratio will be determined in two ways, depending on the application.

Remember that for all combined cycles it is being assumed that the compressor work is equal to the turbine work. This requirement can be expressed by equating (18) and (19):

$$\frac{3T_{13}}{\eta_{comp}}\left(\frac{1}{y^{1/3}\delta} - 1\right) = 2T_4\eta_{turb}\left(1 - \frac{y_{turb}}{\delta}\right) \tag{33}$$

This equation can be written so as to express the compressor inlet temperature and the turbine temperature (and hence pressure) ratio in terms of the other system parameters:

$$\frac{T_{13}}{1 - \frac{y_{turb}}{\delta}} = \frac{2T_4\eta_{turb}\eta_{comp}}{3\left(\frac{1}{y^{1/3}\delta} - 1\right)} \tag{34}$$

Now, specification of either parameter provides a value for the other.

Ground-based system. - For this application, the compressor inlet temperature will be specified directly as the ambient temperature, say 300 K. Then, since the turbine inlet temperature is also assumed, equation (34) provides a value for the turbine stage isentropic temperature ratio

$$y_{turb} = \delta\left[1 - \frac{3T_{13}}{2\eta_{turb}\eta_{comp}T_{14}}\left(\frac{1}{y^{1/3}\delta} - 1\right)\right] \tag{35}$$

The temperature entropy diagram for this cycle is that shown in figure 2(a). The pressures can be written in terms of the isentropic temperature ratio

$$p_a' = p_b' y_{turb}^{\gamma/(\gamma-1)} \qquad p_b' = p_L y_{turb}^{\gamma/(\gamma-1)} \tag{36}$$

Space use. - Here the compressor inlet temperature should be specified to minimize the total system mass. The radiator mass is probably most sensitive to changes in the compressor inlet temperature. Therefore, a logical choice for the compressor inlet temperature is the one determined by that pressure ratio across the turbine which minimizes the radiator mass or area if one assumes that the radiator mass is proportional to the radiator area. The compressor inlet temperature depends (from eq. (34)) on the isentropic temperature ratio (pressure ratio) across the turbine stage. We assume that this ratio is equal to the ratio across the generator, and hence, from equation (16) equal to the ratio across each compressor stage; that is,

$$\frac{p_a'}{p_b'} = \frac{p_a}{p_b} = \left(\frac{p_L}{p_H}\right)^{1/3} \tag{37}$$

Then, using equation (34) the compressor inlet temperature becomes

$$T_{13} = T_4 \frac{2\eta_{turb}\eta_{comp}}{3} \frac{\left(1 - \frac{y^{1/3}}{\delta}\right)}{\left(\frac{1}{\delta y^{1/3}} - 1\right)} \tag{38}$$

The temperature entropy diagram for this cycle is shown in figure 2(b). Using equation (38) it is possible to write equation (32) for the efficiency as

$$\eta = \frac{G}{1 - L_h + GL_h - \frac{T_4}{T_1}\left[\frac{2B}{3C}(1 + C)(1 - \eta_{rec}) + \eta_{rec} - L_h - B(1 + \eta_{rec})\right]} \tag{39}$$

Intercooled Brayton cycle. - Now consider the cycle formed by removing state points 3 to 7 and state point 16. The temperature entropy diagram for the resulting cycle is shown in figure 2(c). This cycle is different from the usual Brayton cycle because of the intercooling between compressor stages. The expander which changes the fluid from temperature 1 to 2 can be either an MHD generator or a gas turbine. In either case, the compressor power must be taken from the power output of the expander.

The power delivered by the expander is

$$P_{exp} = wc_p(T_1 - T_2) \tag{40}$$

The exit temperature can be expressed in terms of the inlet temperature, the expander efficiency, and the pressure ratio. The expander power can then be written as:

$$P_{exp} = wc_p\eta_{exp}T_1\left[1 - \left(\frac{p_L}{p_H - \Delta p_H}\right)^{(\gamma-1)/\gamma}\right] \tag{41}$$

The power required by the compressor is the same as that given by equation (12). The isentropic temperature ratio, y, given in equation (15d) and the fractional pressure loss parameter δ given in equation (14) can be introduced into the expander power equation (41):

$$P_{exp} = wc_p\eta_{exp}T_1\left(1 - \frac{y}{\delta}\right) \tag{42}$$

The compressor power is given in equation (19). A parameter proportional to the work delivered by the expander can be defined.

$$E \equiv \eta_{exp}\left(1 - \frac{y}{\delta}\right) \tag{43}$$

A similar parameter for the compressor is given by equation (20c). The exit temperature from these components can then be expressed in terms of their parameters as

$$T_2 = T_1(1 - E) \tag{44}$$

$$T_{10} = T_{12} = T_{14} = T_{13}(1 + C) \tag{21c}$$

The thermal power from the reactor to the working fluid is

$$Q_{IB} = wc_p(T_1 - T_{15}) \tag{45}$$

The reactor inlet temperature T_{15} can be expressed in terms of previously introduced parameters from equation (24): (where the turbine outlet temperature T_7 is replaced by the expander outlet temperature T_2):

$$T_{15} = \eta_{rec}T_2 + (1 - \eta_{rec})T_{14} \tag{46}$$

Equation (46) can be written after substitution for T_2 from equation (44) and for T_{14} from equation (21c) as

$$T_{15} = \eta_{rec}T_1(1 - E) + T_{13}(1 - \eta_{rec})(1 + C) \tag{47}$$

The thermal power (eq. 45) then becomes

$$Q_{IB} = wc_p\left[T_1 - \eta_{rec}(1 - E)T_1 - (1 - \eta_{rec})(1 + C)T_{13}\right] \tag{48}$$

The overall thermodynamic cycle efficiency is defined as

$$\eta_{IB} = \frac{P_{exp} - P_{comp}}{Q_{IB}} = \frac{T_1 - T_2 - 3(T_{14} - T_{13})}{T_1 - T_{15}} \tag{49}$$

and after substitution for P_{exp}, P_{comp} and Q_{IB} become:

$$\eta_{IB} = \frac{E - 3\dfrac{T_{13}}{T_1}C}{1 - (1 - E)\eta_{rec} - \dfrac{T_{13}}{T_1}(1 + C)(1 - \eta_{rec})} \tag{50}$$

The ratio of compressor inlet to reactor exit temperature remains to be specified.

Specific Radiator Area

From the schematic diagram of figure 1 it is apparent that there are three heat exchangers requiring radiator area: two intercooling heat exchangers and the primary cooler. The temperature-entropy diagrams of figure 2 indicate that, conceptually, the total area required by these heat exchangers is equivalent to providing three radiators whose total area reduces the working fluid temperature from T_{14} to T_{13} and one radiator whose area is sufficient to reduce the working fluid from temperature T_8 to T_{14}. If one neglects the temperature difference between the working fluid and the radiator surface and assumes the radiator radiates to a sink at absolute zero, the total area for each intercooling radiator becomes (ref. 3):

$$A_{int} = \frac{wc_p}{3\epsilon\sigma}\left(\frac{1}{T_{13}^3} - \frac{1}{T_{14}^3}\right) \tag{51}$$

and the area for the primary cooler radiator is

$$A_{pri} = \frac{wc_p}{3\epsilon\sigma}\left(\frac{1}{T_{13}^3} - \frac{1}{T_{14}^3} + \frac{1}{T_{14}^3} - \frac{1}{T_8^3}\right) \tag{52}$$

so that the total radiator area is the sum of the three radiator areas:

$$A_{rad} = \frac{wc_p}{\epsilon\sigma T_{13}^3}\left[\left(1 - \frac{T_{13}^3}{T_{14}^3}\right) + \frac{1}{3}\frac{T_{13}^3}{T_{14}^3}\left(1 - \frac{T_{14}^3}{T_8^3}\right)\right] \tag{53}$$

The temperature ratio T_{13}/T_{14} can be eliminated using equation (21c). Then equation (53) becomes

$$A_{rad} = \frac{wc_p}{\epsilon\sigma T_{13}^3}\left[1 - \frac{1}{(1 + C)^3} + \frac{1}{3(1 + C)^3}\left(1 - \frac{T_{14}^3}{T_8^3}\right)\right] \tag{54}$$

The specific radiator area is defined as the total radiator area divided by the net electric power output.

Combined cycle. - The net electric power for the combined cycle can be written using equations (17) and (20a) as

$$P_{gen} = P_e = wc_pT_1G \tag{55}$$

The primary radiator inlet temperature T_8 can be expressed (using equation (24)) as

$$T_8 = \eta_{rec} T_{14} + (1 - \eta_{rec}) T_7 \tag{56}$$

and rewritten using equations (21b), (21c), and (38) as

$$\frac{T_8}{T_{14}} = \eta_{rec} + (1 - \eta_{rec}) \frac{3C}{2B} \frac{(1 - B)}{(1 + C)} \tag{57}$$

The specific radiator area can now be written by substituting equation (57) into (54) and dividing by equation (55):

$$\frac{A_{rad}}{P_e} = \frac{1}{\epsilon \sigma G T_1} \left(\frac{3C}{2BT_4}\right)^3$$

$$\times \left(1 - \frac{1}{(1 + C)^3} + \frac{1}{3(1 + C)^3} \left\{1 - \left[\eta_{rec} + (1 - \eta_{rec}) \frac{3C}{2B} \frac{(1 - B)}{(1 + C)}\right]^{-3}\right\}\right) \tag{58}$$

The specific radiator area becomes smaller at higher radiator temperatures. A convenient nondimensionalization for the specific radiator area is the value it would have if the radiator were operated at the maximum temperature in the cycle T_1. Using this temperature, a specific radiator area parameter α can be defined:

$$\alpha \equiv \frac{A_{rad}}{P_e} \epsilon \sigma T_1^4 \tag{59}$$

This definition can be used to rewrite equation (58) as

$$\alpha = \left(\frac{3CT_1}{2BT_4}\right)^3 \frac{1}{G} \left\{1 - \frac{2}{3(1 + C)^3} - \frac{1}{3}\left[\frac{2B}{2B(1 + C)\eta_{rec} + (1 - \eta_{rec})(1 - B)3C}\right]^3\right\} \tag{60}$$

At this point a comment about the assumption that $y_t = y^{1/3}$ can be made. From equation (60) it is possible to calculate α as a function of y_t (using eqs. (20a) and (20b)) for typical values of the other parameters and to compare the minimum value with the values for $y_t = y^{1/3}$. For a case where $\eta_{rec} = 1.0$, $\Delta p/p = 0.015$ and $p_L/p_H = 0.125$, the minimum y_t differs from $y^{1/3}$ by 3 percent, and the α differs from the minimum by about 5 percent.

Intercooled Brayton. - The net electric power generated by the intercooled Brayton cycle is given by the difference between equation (42) and equation (19):

$$P_e = P_{exp} - P_{comp} = wc_pT_1\left[\eta_{exp}\left(1 - \frac{y}{\delta}\right) - \frac{3}{\eta_{comp}}\frac{T_{13}}{T_1}\left(\frac{1}{y^{1/3}\delta} - 1\right)\right] \tag{61}$$

The primary radiator inlet temperature T_8 can be expressed in terms of the system parameters. Replace the turbine outlet temperature T_7 by the expander outlet temperature T_2 in equation (24) and solve for T_8:

$$T_8 = \eta_{rec}T_{14} + (1 - \eta_{rec})T_2 \tag{62}$$

This equation may be rewritten using equations (44) and (21c):

$$\frac{T_8}{T_{14}} = \eta_{rec} + (1 - \eta_{rec})\frac{T_1}{T_{13}}\frac{(1 - E)}{(1 + C)} \tag{63}$$

The specific radiator area for the intercooled Brayton cycle can now be expressed by dividing equation (54) by equation (61) and substituting for T_{14}/T_8 from equation (63)

$$\frac{A_{rad}}{P_e} = \frac{1 - \dfrac{1}{(1 + C)^3} + \dfrac{1}{3(1 + C)^3}\left\{1 - \left[\eta_{rec} + (1 - \eta_{rec})\dfrac{T_1(1 - E)}{T_{13}(1 + C)}\right]^{-3}\right\}}{\epsilon\sigma T_{13}^3\left[T_1\eta_{exp}\left(1 - \dfrac{y}{\delta}\right) - T_{13}\dfrac{3}{\eta_{comp}}\left(\dfrac{1}{y^{1/3}\delta} - 1\right)\right]} \tag{64}$$

The specific radiator parameter defined in equation (59) can be rewritten after substituting equations (44) and (21c) into equation (64):

$$\alpha_{IB} = \left(\frac{T_1}{T_{13}}\right)^3 \frac{1 - \dfrac{2}{3(1 + C)^3} - \dfrac{1}{3}\left[\dfrac{1}{(1 + C)\eta_{rec} + (1 - \eta_{rec})\dfrac{T_1}{T_{13}}(1 - E)}\right]^{-3}}{E - \dfrac{T_{13}}{T_1}3C} \tag{65}$$

The ratio of compressor inlet to reactor exit temperature T_{13}/T_1 remains to be specified, just as in the case of the cycle efficiency (eq. (50)). This ratio will be specified as that value which minimizes the specific radiator area parameter.

Recuperator Area

The individual heat exchanger area required depends on the associated temperature difference. For the combined cycle there are two recuperators and one turbine reheat heat exchanger required. We will assume that the temperature difference across the high-temperature recuperator and the turbine reheat heat exchanger is much greater than that across the low-temperature recuperator. The area of these heat exchangers will be neglected in comparison with the area of the low-temperature recuperator. There is only one recuperator for the intercooled Brayton cycle. The low-temperature recuperator area for both cycles will be the same.

The logarithmic mean temperature difference across the recuperator is

$$\Delta T = \frac{(T_7 - T_{15}) - (T_8 - T_{14})}{\ln \dfrac{T_7 - T_{15}}{T_8 - T_{14}}} \tag{66}$$

(and T_7 is replaced by T_2 for the intercooled Brayton cycle). But it is assumed that there is no heat lost to the environment, so the logarithmic mean temperature difference, in the limit where $(T_7 - T_8) \rightarrow (T_{15} - T_{14})$, becomes

$$\Delta T = \frac{(T_7 - T_{15}) + (T_8 - T_{14})}{2} \tag{67}$$

Substituting for T_{14} and T_{15} from equation (24), equation (67) becomes

$$\Delta T = \frac{1 - \eta_{rec}}{\eta_{rec}} (T_7 - T_8) \tag{68}$$

The heat transferred can also be written as

$$Q_{rec} = wc_p(T_7 - T_8) = UA_{rec}\Delta T \tag{69}$$

Thus, the recuperator area is

$$A_{rec} = \frac{wc_p}{U} \frac{\eta_{rec}}{1 - \eta_{rec}} \tag{70}$$

The overall heat-transfer coefficient U must be determined. If the coefficient for the outside and the inside of the recuperator tubes are assumed to be equal (to h), then

$$U = \frac{h}{2} \tag{71}$$

This film coefficient can now be expressed in terms of the friction factor for turbulent flow by using Reynold's analogy (as modified by Colburn and given in Jakob (ref. 4)):

$$h = \frac{f}{2} \frac{\rho u c_p}{(N_{Pr})^{2/3}} \tag{72}$$

Then, equation (70) becomes

$$A_{rec} = \frac{\eta_{rec}}{1 - \eta_{rec}} \frac{4w(N_{Pr})^{2/3}}{\rho u f} \tag{73}$$

The fluid velocity through the heat exchanger can be expressed in terms of the friction factor:

$$A_{cs} \frac{\Delta p}{p} = f \frac{u^2}{2RT} A_{rec} \tag{74}$$

and the mass flow through the recuperator is

$$\rho u A_{cs} = w \tag{75}$$

Therefore, equations (73), (74), and (75) can be combined

$$\frac{\Delta p}{p} = \frac{\eta_{rec}}{1 - \eta_{rec}} N_{Pr}^{2/3} \frac{2u^2}{RT} \tag{76}$$

The recuperator area can be written by solving for the velocity from equation (76) and substituting it into equation (73):

$$A_{rec} = \left(\frac{\eta_{rec}}{1 - \eta_{rec}}\right)^{3/2} \frac{4\sqrt{2}\, w\, N_{Pr} \sqrt{RT}}{fp\sqrt{\frac{\Delta p}{p}}} \tag{77}$$

The density has been eliminated by using the perfect gas law. A typical value of the recuperator pressure is required. We will use the average pressure,

$$p = \frac{1}{2}(p_L + p_H) \tag{78}$$

The specific recuperator area is defined as the total area divided by the net electrical power.

Combined cycle. - The net power for the combined cycle is given in equation (55):

$$P_e = wc_p T_1 G \tag{55}$$

A representative temperature in the recuperator is required. The representative value will be taken as the average:

$$T = \frac{T_7 + T_8 + T_{15} + T_{14}}{4} \tag{79}$$

The average temperature can be rewritten using equation (24) as

$$T = \frac{1}{2}(T_7 + T_{14}) \tag{80}$$

which can be rewritten using equations (21b), and (21c), and (38) as

$$T = T_4\left[\frac{(1 - B)}{2} + \frac{B}{3C}(1 + C)\right] \tag{81}$$

The specific recuperator area can now be written by substituting for T from equation (81) into (77) and dividing by (55):

$$\frac{A_{rec}}{P_e} = \left(\frac{\eta_{rec}}{1 - \eta_{rec}}\right)^{3/2} \frac{\gamma - 1}{1} \frac{8\sqrt{2}\, N_{Pr} \sqrt{RT_4} \left[\frac{1 - B}{2} + \frac{B}{3C} (1 + C)\right]^{1/2}}{fp_H \left(1 + \frac{p_L}{p_H}\right) \sqrt{\frac{\Delta p}{p}\, RT_1 G}} \tag{82}$$

A specific recuperator area parameter $\mathscr{R}$ will now be introduced, where

$$\mathscr{R} \equiv \frac{A_{rec}}{P_e} \cdot \frac{\gamma}{\gamma - 1} \frac{p_H \sqrt{RT_1}}{8\sqrt{2}} \frac{f}{N_{Pr}} \tag{83}$$

Then equation (82) can be written as

$$\mathscr{R} = \frac{\sqrt{\frac{T_4}{T_1}} \sqrt{\frac{1 - B}{2} + \frac{(1 + C)}{C} \frac{B}{3}}}{\sqrt{\frac{\Delta p}{p}}\, G \left(1 + \frac{p_L}{p_H}\right)} \left(\frac{\eta_{rec}}{1 - \eta_{rec}}\right)^{3/2} \tag{84}$$

Intercooled Brayton cycle. - The net electrical power for this cycle is given by equation (61):

$$P_e = wc_pT_1 \left[\eta_{exp} \left(1 - \frac{y}{\delta}\right) - \frac{3}{\eta_{comp}} \frac{T_{13}}{T_1} \left(\frac{1}{y^{1/3}\delta} - 1\right)\right] \tag{61}$$

Again the representative recuperator temperature will be taken as the average

$$T = \frac{1}{2} (T_2 + T_8) \tag{85}$$

This temperature can be written, using equations (44) and (21c) as

$$T = \frac{T_1}{2} \left[1 - E + \frac{T_{13}}{T_1} (1 + C)\right] \tag{86}$$

The specific recuperator area for this cycle can be written by substitution for T from equation (86) into (77) and dividing by (61):

$$\frac{A_{rec}}{P_e} = \left(\frac{\eta_{rec}}{1 - \eta_{rec}}\right)^{3/2} \cdot \frac{\gamma - 1}{\gamma} \cdot \frac{8\sqrt{2}\, N_{Pr} \left(\frac{1 - E}{2} + \frac{T_{13}}{T_1}\frac{1 + C}{2}\right)^{1/2}}{fp_H\left(1 + \frac{p_L}{p_H}\right)\sqrt{\frac{\Delta p}{p}}\sqrt{RT_1}\left(E - \frac{T_{13}}{T_1}3C\right)} \tag{87}$$

The temperature ratio T_{13}/T_1 is that value which provides the minimum value of α_{IB}. Using the specific recuperator area parameter given in equation (83), equation (87) can be written

$$\mathscr{R}_{IB} = \frac{\sqrt{\frac{1 - E}{2} + \frac{T_{13}}{T_1}\frac{(1 + C)}{2}}}{\sqrt{\frac{\Delta p}{p}}\left(E - 3C\,\frac{T_{13}}{T_1}\right)\left(1 + \frac{p_L}{p_H}\right)}\left(\frac{\eta_{rec}}{1 - \eta_{rec}}\right)^{3/2} \tag{88}$$

Heat Exchanger and Radiator Mass

For space application, it is important to minimize the mass of both the radiator and the recuperator. The total specific mass of these two components can be defined as the total mass divided by the net electric power output and can be written as:

$$\frac{m}{P_e} = m_{rad}\frac{A_{rad}}{P_e} + m_{rec}\frac{A_{rec}}{P_e} \tag{89}$$

Substituting for the specific areas from equations (59) and (83) results in

$$\frac{m}{P_e} = m_{rad}\frac{\alpha}{\epsilon\sigma T_1^4} + \frac{\gamma - 1}{\gamma}\,\frac{8\sqrt{2}\,N_{Pr}}{p_H\sqrt{RT_1}\,f}\,m_{rec}\mathscr{R} \tag{90}$$

This specific mass is obviously the weighted sum of the radiator and recuperator specific area parameters. The specific mass can be nondimensionalized by using the radiator specific mass. A specific mass parameter $\mathscr{M}$ can be defined:

$$\mathscr{M} \equiv \frac{\epsilon\sigma T_1^4 m}{m_{rad}P_e} \tag{91}$$

and equation (90) can be rewritten using equation (91)

$$\mathscr{M} = \alpha + r\mathscr{R} \tag{92}$$

where

$$r \equiv \frac{(\gamma - 1)}{\gamma} \frac{8\sqrt{2}\, N_{Pr} m_{rec}}{p_H f \sqrt{RT_1}\, m_{rad}} \epsilon \sigma T_1^4 \tag{93}$$

The parameter r represents the contribution of the recuperator specific mass to the total specific mass. If $r = 0$; then minimum $\mathscr{M}$ corresponds to minimum α. If $r = 1$, the recuperator and radiator contribute about equally to the total specific mass, and for large r the specific radiator mass becomes less important.

RESULTS AND DISCUSSION

Specification of Parameters

Compressor and turbine efficiency. - It is advantageous to use all cycle components at their maximum efficiency. Therefore, for all calculations a reasonably high value of compressor and turbine efficiency is chosen. These values are 0.88 for the compressor and 0.90 for the turbine (ref. 5).

Recuperator effectiveness. - It would be desirable to have a value as high as possible for the recuperator effectiveness. However, some temperature drop is required in order to get the heat transfer. There are two important parameters in the design and selection of heat exchangers: the effectiveness and the pressure drop. In this report we will fix the value of effectiveness at 0.8 (unless otherwise noted) and determine the implications of this selection on the pressure drop. The sensitivity of the overall thermodynamic cycle efficiency to changes in the recuperator effectiveness will be calculated for the ground based application in order to show the importance of this parameter.

High-temperature heat exchangers. - The high-temperature recuperator and turbine reheat heat exchanger are characterized differently from the low-temperature recuperator. A loss parameter L_h is specified which represents the fraction of the heat taken from the hot working fluid which is transferred to the cool working fluid. It seems reasonable to assume $L_h = 0.95$. The overall cycle efficiencies are not too sensitive to the value of this parameter ($\partial\eta/\partial L_h \sim 1/2$ for a typical set of conditions).

Generator efficiency. - Two types of working fluids are envisioned for the MHD generator: one which is seeded with cesium and has sufficient equilibrium ionization and electrical conductivity, and one seeded with xenon and requires nonequilibrium ionization. If this nonequilibrium ionization is provided by the Hall effect and electrode segmentation, then a larger generator current density would be required than in the equilibrium case. This larger current density would result in a lower efficiency. Reasonable generator efficiency upper limits for these two working fluids may be 80 percent for cesium and 55 percent for xenon. These two numbers will be used herein.

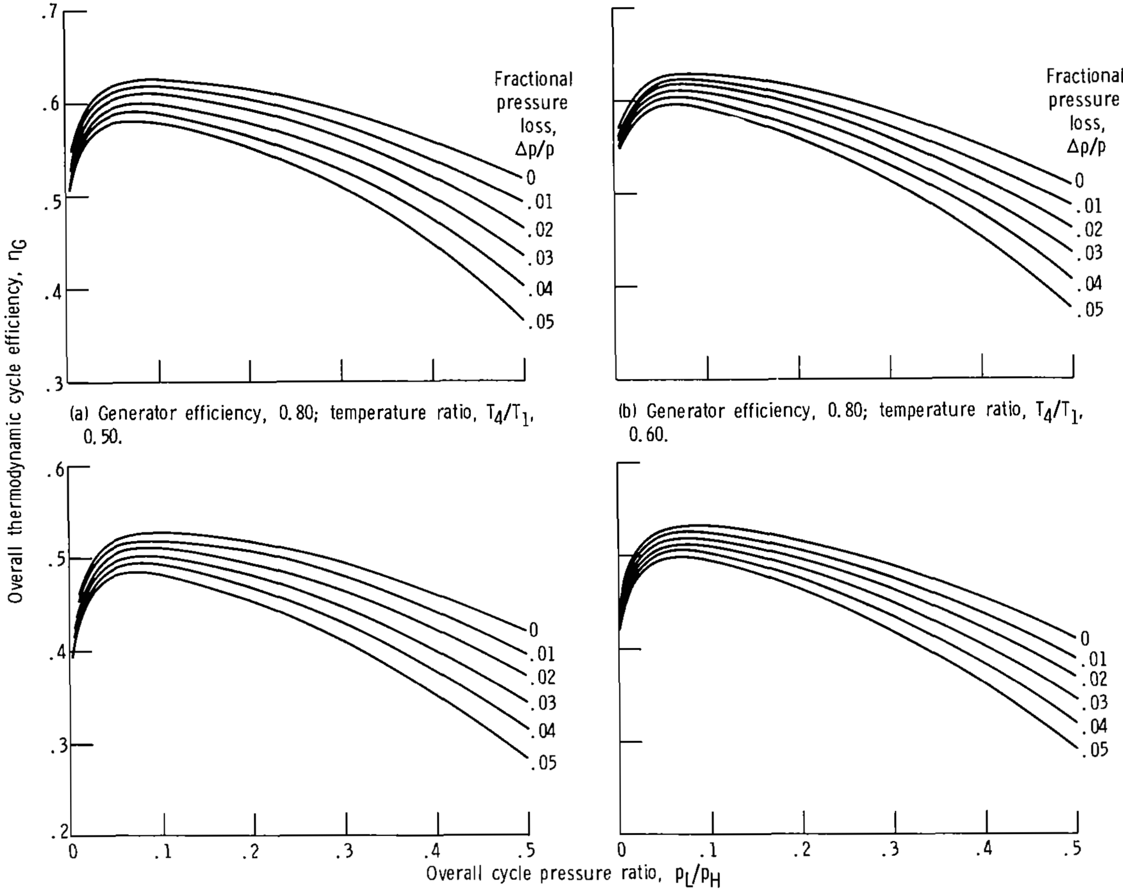
Maximum cycle temperature. - It is advantageous to have a high reactor-outlet temperature. The temperature of the hydrogen leaving the NERVA reactor is about 2500 K. We will assume this temperature for all cycles using an MHD generator. We will consider two cycles with turbines only, and those cycles will have reactor outlet temperatures of 1250 and 1500 K.

Turbine inlet temperature. - It is advantageous to have a high turbine-inlet temperature. We have chosen two values for this temperature, 1250 and 1500 K, which are expressed as 50 and 60 percent of the maximum cycle temperature, respectively. These values seem to be the conservative and optimistic values, respectively, for uncooled turbines operating in an inert gas.

Overall Thermodynamic Cycle Efficiency

Combined cycle - ground-based use. - The efficiencies calculated from equation (32) for the selected values of turbine-inlet to reactor-outlet temperature ratios and generator efficiencies are shown in figure 3. The turbine isentropic pressure ratio is determined from equation (35) with the compressor inlet to reactor outlet temperature ratio fixed at 0.12 (T_{13} = 300 K when T_1 = 2500 K). The results show at least one feature which should be discussed. There is, in all cases, an overall cycle pressure ratio that maximizes the efficiency. The calculation at pressure ratios below that value are not valid because the fluid temperature leaving the generator is too low to provide the required turbine reheat. Generally speaking, the efficiency increases with increasing generator efficiency and increasing turbine-inlet temperature ratio, and decreases with increasing overall cycle pressure ratio and fractional pressure drop. The results of other analyses are compared with those of this analysis in appendix B.

The effect of recuperator effectiveness on the overall thermodynamic cycle efficiency for the ground based combined cycle is shown in figure 4. The overall efficiency is quite sensitive to recuperator effectiveness. The value of recuperator effectiveness which is achieved is related to pressure drop and recuperator size. As a matter of fact, a recuperator effectiveness increase from 0.8 to 1.0 does not improve the cycle efficiency if the pressure drop simultaneously increases from 0.01 to 0.1. High recuperator effectiveness at low pressure drop means large recuperators. The penalty that must be paid



(a) Generator efficiency, 0.80; temperature ratio, T_4/T_1, 0.50.

(b) Generator efficiency, 0.80; temperature ratio, T_4/T_1, 0.60.

(c) Generator efficiency, 0.55; temperature ratio, T_4/T_1, 0.50.

(d) Generator efficiency, 0.55; temperature ratio, T_4/T_1, 0.60.

Figure 3. - Overall thermodynamic combined cycle efficiency for ground-based use as function of overall cycle pressure ratio. Recuperator effectiveness, 0.80; minimum to maximum temperature ratio, 0.12.

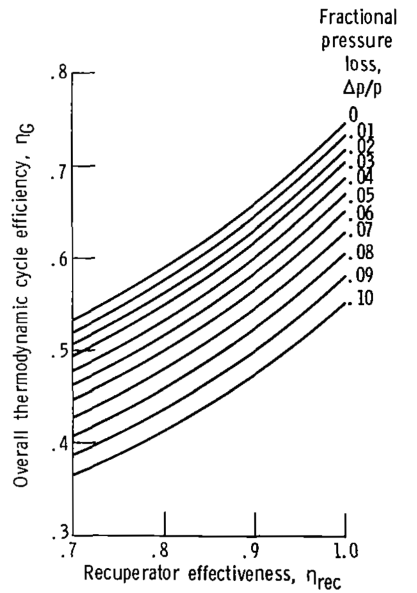


Figure 4. - Overall thermodynamic cycle efficiency as function of recuperator effectiveness for combined cycle. Overall cycle pressure ratio, 0.3; generator efficiency, 0.8. Temperature ratios: T_4/T_1 = 0.60, T_{13}/T_1 = 0.12.

depends on the application. Others have chosen values of 0.78 (ref. 2), 0.94 (ref. 4), and 0.91 (ref. 5) (see appendix B). An exact assessment for any given application requires a more thorough study than will be undertaken here. Suffice it to say that recuperator effectiveness is a very sensitive parameter for the Brayton cycle system.

Combined cycle - space use. - The efficiency is calculated from equation (39) and shown in figure 5 for the same conditions as those assumed for figure 3. Generally, the efficiency behaves the same as that found for the ground-based application, except that the space-use efficiency decreases with increasing turbine inlet temperature ratio. Because the compressor power is assumed to equal the turbine power, the compressor-inlet temperature increases as the turbine-inlet temperature increases (as shown in

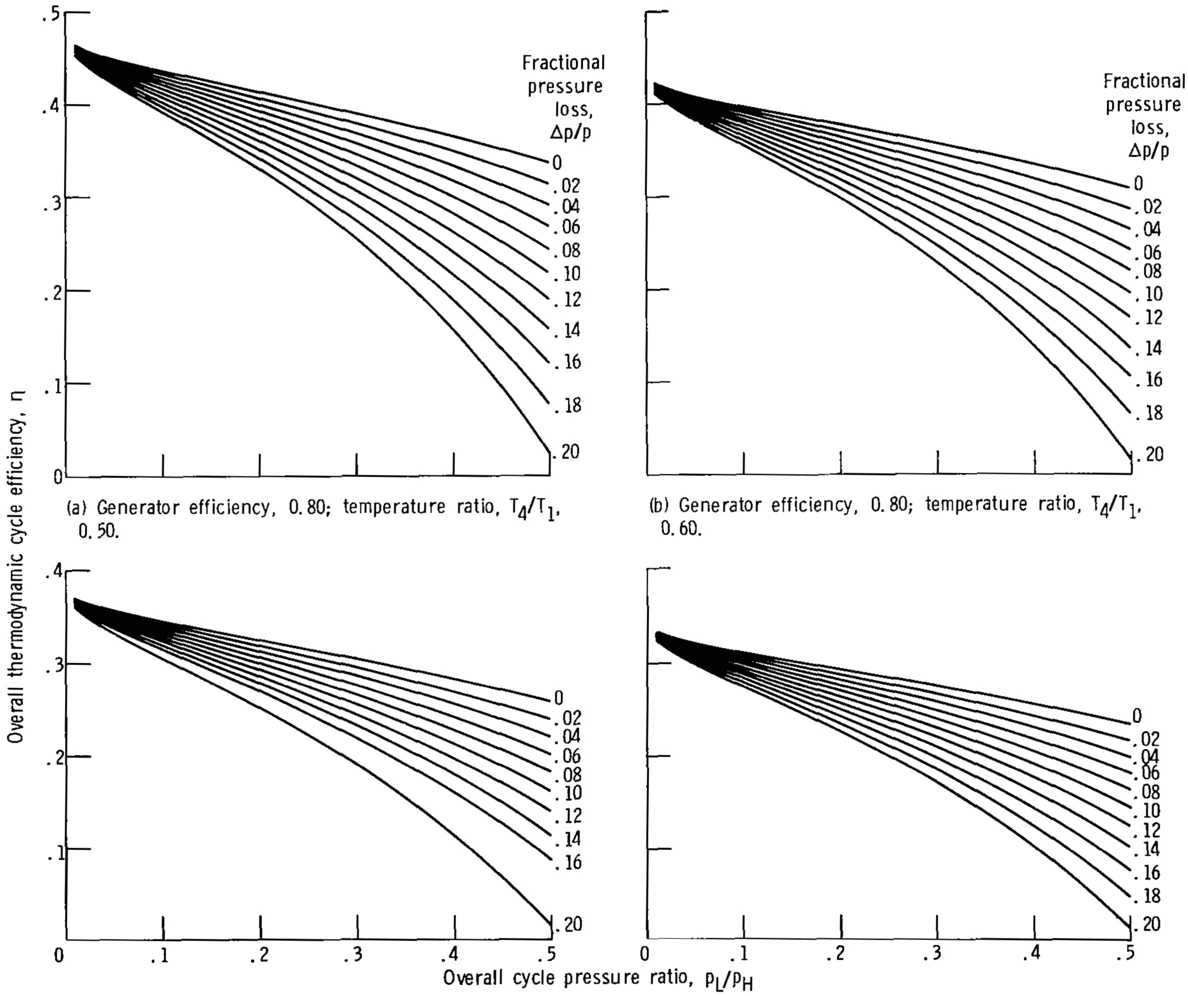


Figure 5. - Overall thermodynamic combined cycle efficiency as function of overall cycle pressure ratio. Recuperator effectiveness, 0.8.

eq. (38)). This increase in the minimum cycle temperature results in reduced efficiency.

Intercooled Brayton cycle. - The efficiency for this cycle is calculated from equation (50) for expander efficiencies of 55, 80, and 90 percent. These correspond to the case when the expander is an MHD generator (the first 2) and a turbine (the last). The ratio of compressor inlet to reactor outlet temperature is chosen as that value which minimizes α_{IB} in equation (65) for the appropriate value of expander efficiency. This ratio is plotted in figure 6, and the resulting efficiency is shown in figure 7. The temperature ratio has a maximum value for certain fractional pressure drops in the overall cycle pressure ratio region under consideration. This temperature is determined by the value for which the increased net power resulting from expansion to a lower temperature is offset by the resulting radiator area increase. At the higher overall cycle pressure ratios, the effect of pressure losses is more severe on generator performance than on radiator area. Therefore, a slightly lower temperature will increase the generator output much more than it will increase the radiator area. The efficiency increases with expander efficiency and decreases with increasing overall cycle pressure ratio and fractional pressure loss. The results of other Brayton cycle studies are compared with the results of this analysis in appendix B.

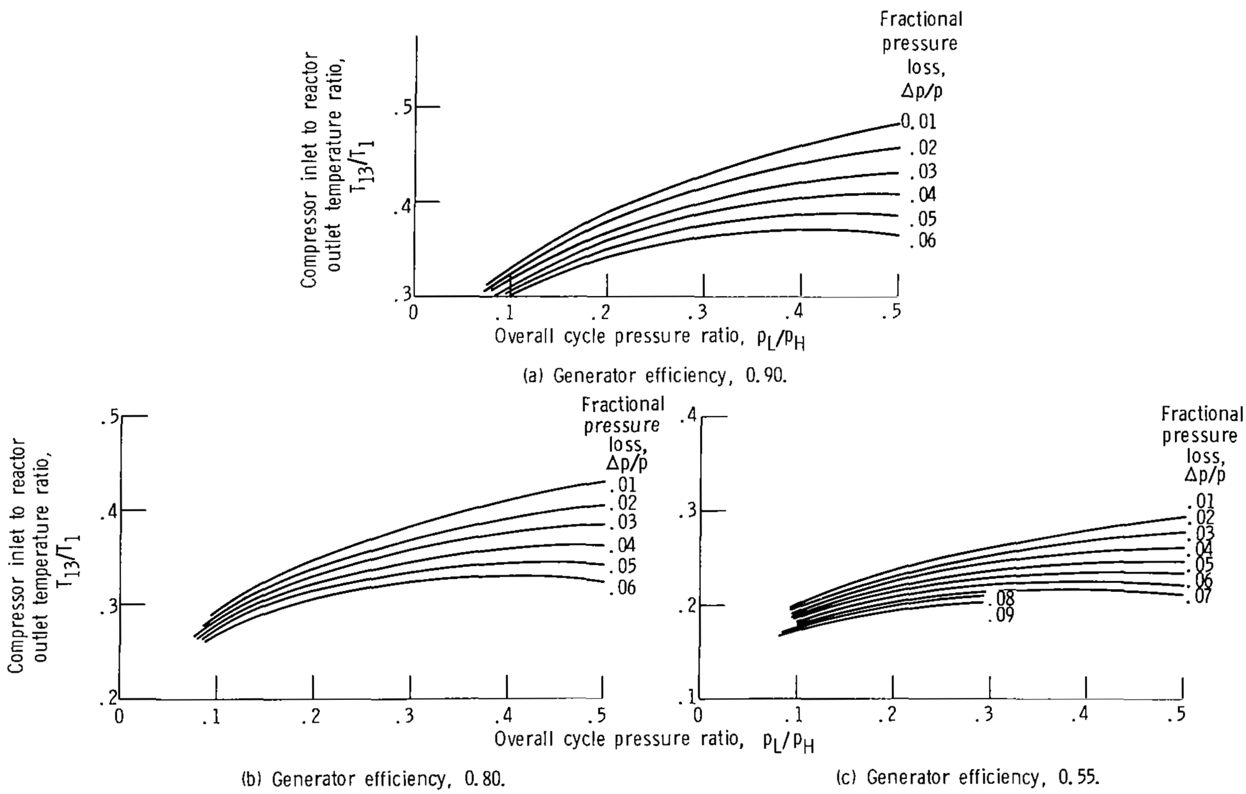


Figure 6. - Ratio of compressor inlet to reactor outlet temperature which minimizes specific radiator parameter for intercooled Brayton cycle plotted as function of overall cycle pressure ratio. Recuperator effectiveness, 0.80.

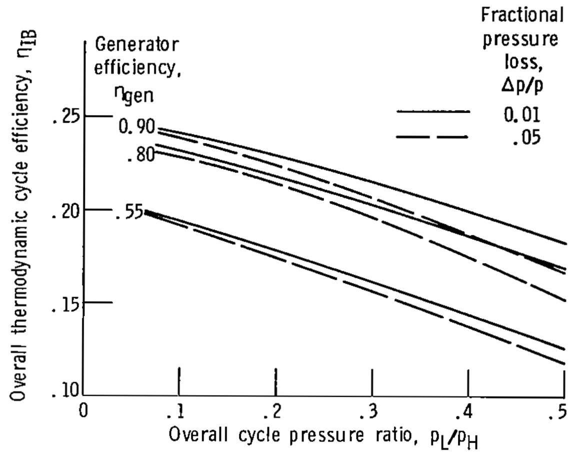
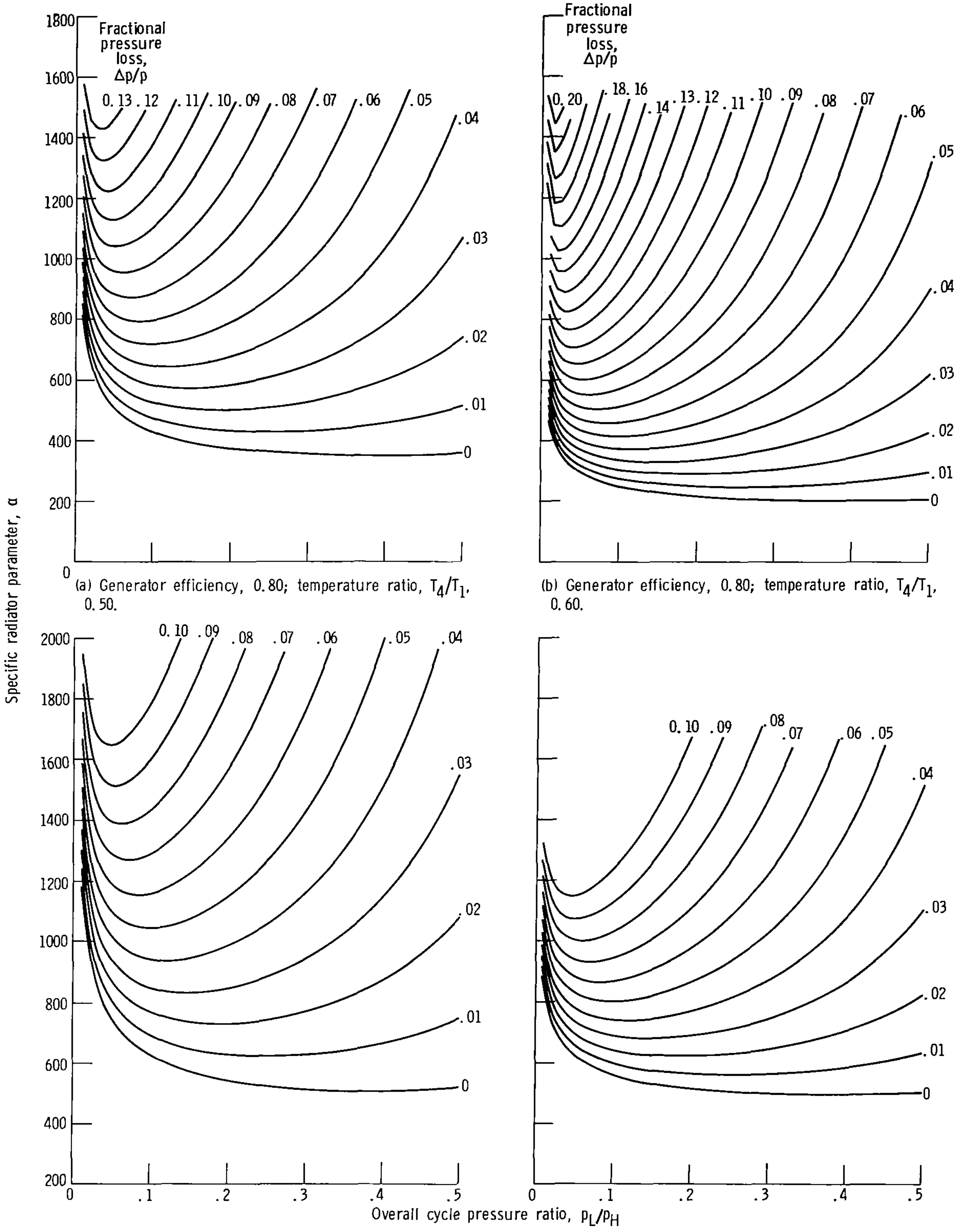


Figure 7. - Overall thermodynamic intercooled Brayton cycle efficiency plotted as function of overall cycle pressure ratio. The ratio of compressor inlet to reactor outlet temperatures was chosen to minimize the specific radiator area parameter. Recuperator effectiveness, 0.8.

Specific Radiator Area Parameter

Combined cycle - space use. - The specific radiator area parameter can be calculated from equation (60) for the same conditions as shown in figure 5. The results are shown in figure 8. It should first be noted that for a fixed fractional pressure loss parameter there is a value of overall pressure ratio that minimizes the specific radiator area parameter. Second, this minimum value increases with increasing pressure loss. The specific radiator area parameter also generally decreases with increasing generator efficiency and increasing turbine-inlet temperature ratio.

Intercooled Brayton. - The specific radiator area parameter can be calculated from equation (65) for the intercooled Brayton cycle. Remember that the compressor inlet temperature has already been determined to minimize the specific radiator parameter in order to determine the overall thermodynamic cycle efficiency. The specific radiator area parameter is calculated at the same conditions as figures 6 and 7 and is shown in figure 9. Again, as for the combined cycle, notice that for a fixed fractional pressure loss there is a value of overall cycle pressure ratio, which provides a minimum value of the specific radiator area parameter. Second, this minimum value increases with increasing fractional pressure loss. At a fixed value of overall cycle pressure ratio and fractional pressure loss, the specific radiator area parameter increases with decreasing generator efficiency.



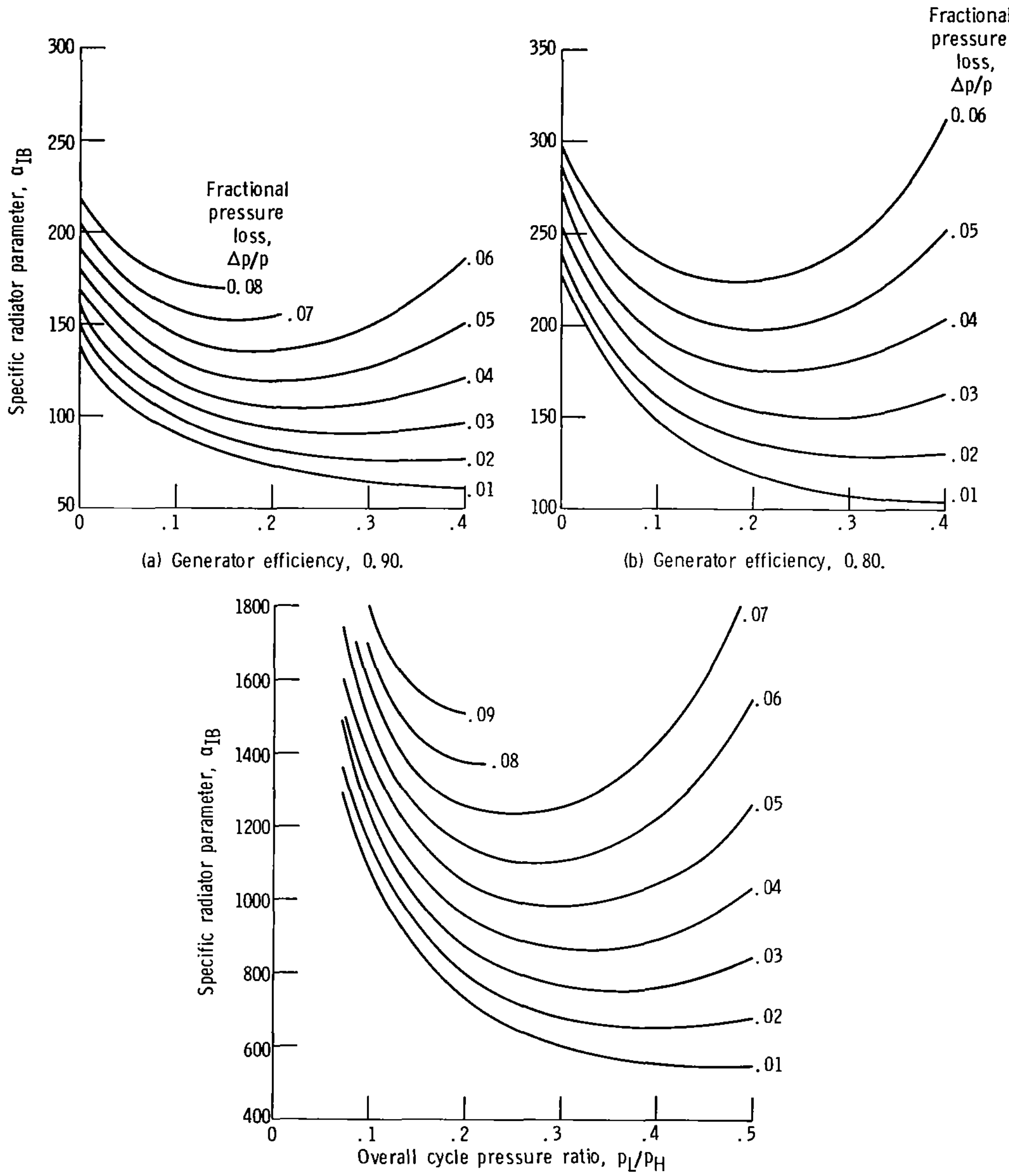
(a) Generator efficiency, 0.80; temperature ratio, T_4/T_1, 0.50.

(b) Generator efficiency, 0.80; temperature ratio, T_4/T_1, 0.60.

(c) Generator efficiency, 0.55; temperature ratio, T_4/T_1, 0.50.

(d) Generator efficiency, 0.55; temperature ratio, T_4/T_1, 0.60.

Figure 8. - Specific radiator area parameter for combined cycle plotted as function of overall cycle pressure ratio. Recuperator effectiveness, 0.80.



(c) Generator efficiency, 0.55.

Figure 9. - Specific radiator area parameter for intercooled Brayton cycle as function of overall cycle pressure ratio. Recuperator effectiveness, 0.80.

Specific Recuperator Area

Combined cycle. - The specific recuperator area is calculated from equation (84) and is shown in figure 10 for the same conditions as in figure 5. This parameter increases with increasing overall cycle pressure ratio, decreasing fractional pressure loss, increasing turbine inlet temperature, and decreasing generator efficiency.

Intercooled Brayton cycle. - The specific recuperator area is calculated from equation (88) for the compressor inlet temperature and other conditions of figure 6, and shown in figure 11. This parameter increases with increasing overall cycle pressure ratio, decreasing pressure loss parameter, and decreasing generator efficiency.

Total Radiator and Recuperator Specific Mass

From figures 8 and 10 the specific radiator area parameter for the combined cycle can be replotted as a function of the specific recuperator parameter. The results are shown in figure 12(a) for parts (a) and (c) of figures 8 and 10 and in figure 12(b) for parts (b) and (d) of figures 8 and 10. These curves show that there is an envelop that minimizes the function $\mathscr{M} = \alpha + r\mathscr{R}$. The exact values of α and $\mathscr{R}$ which provide the minimum depend on r. For comparison of the four cycles of figure 12, a value of $r = 1$ was chosen. This corresponds to Neon at 10^6 N/m^2, $m_{rad} = 8.4$ kilograms per square meter (ref. 10), $m_{rec} = 3$ kilograms per square meter (ref. 9), $\epsilon = 0.9$, $T_1 = 2500$ K, and $f = 0.0025$.

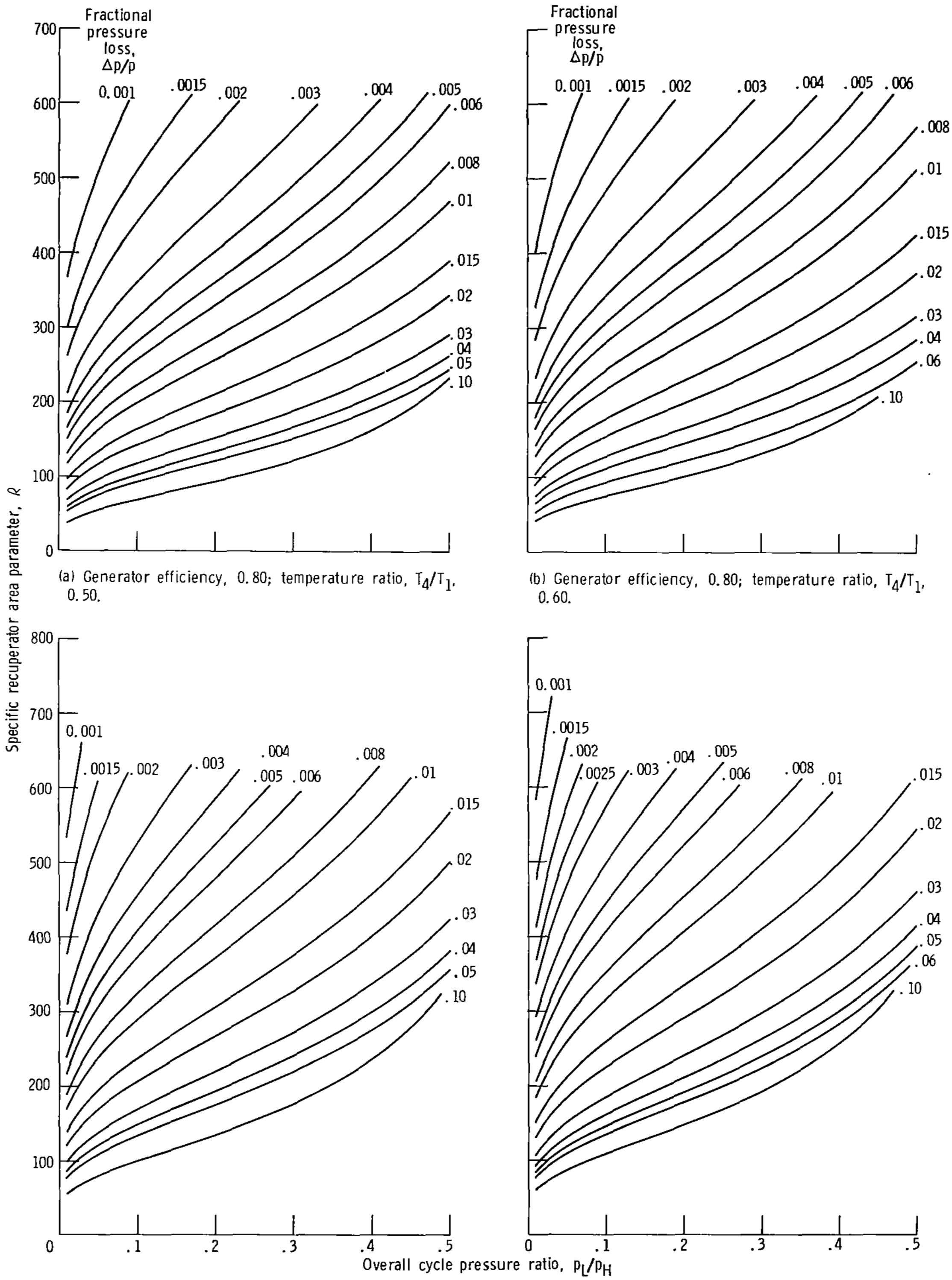
Similar cross-plots of specific radiator area parameter as a function of specific recuperator area parameter can be made from figures 9 and 11 for the intercooled Brayton cycle. These curves are shown in figure 13. Again, the envelope provides the minimum value of $\alpha_{IB} + r\mathscr{R}_{IB}$. In order to compare these intercooled Brayton MHD systems with the turbo-MHD systems, a value of $r = 1$ was again chosen.

For the intercooled Brayton turbine system, however, the temperature T_1 is not 2500°. In order to compare the total mass, the parameter r must be adjusted.

From equation (93), if one calculates $r = (T_1/2500)^{3.5}$, then the relative importance of recuperator and radiator specific masses will be maintained. Finally for all systems, a representative specific total mass can be calculated from equation (91)

$$\frac{m}{P_e} = \frac{m_{rad}}{\epsilon\sigma T_1^4}\mathscr{M} \qquad (91)$$

where $\mathscr{M}$ will be chosen as the minimum value described above.



(a) Generator efficiency, 0.80; temperature ratio, T_4/T_1, 0.50.

(b) Generator efficiency, 0.80; temperature ratio, T_4/T_1, 0.60.

(c) Generator efficiency, 0.55; temperature ratio, T_4/T_1, 0.50.

(d) Generator efficiency, 0.55; temperature ratio, T_4/T_1, 0.60.

Figure 10. - Specific recuperator area parameter for combined cycle plotted as function of overall cycle pressure ratio. Recuperator effectiveness, 0.80.

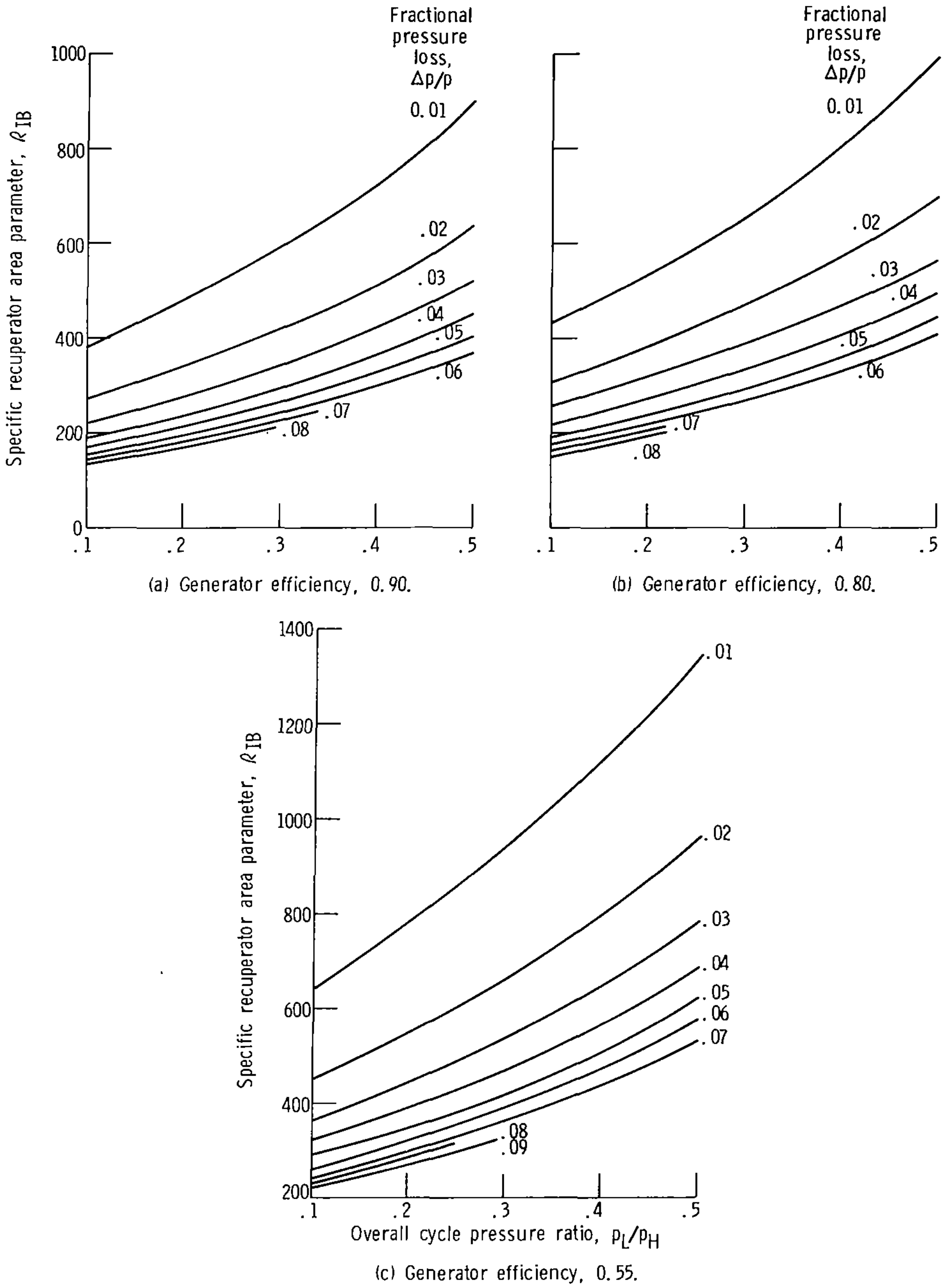


Figure 11. - Specific recuperator area parameter for intercooled Brayton cycle plotted as function of overall cycle pressure ratio. Recuperator effectiveness, 0.80.

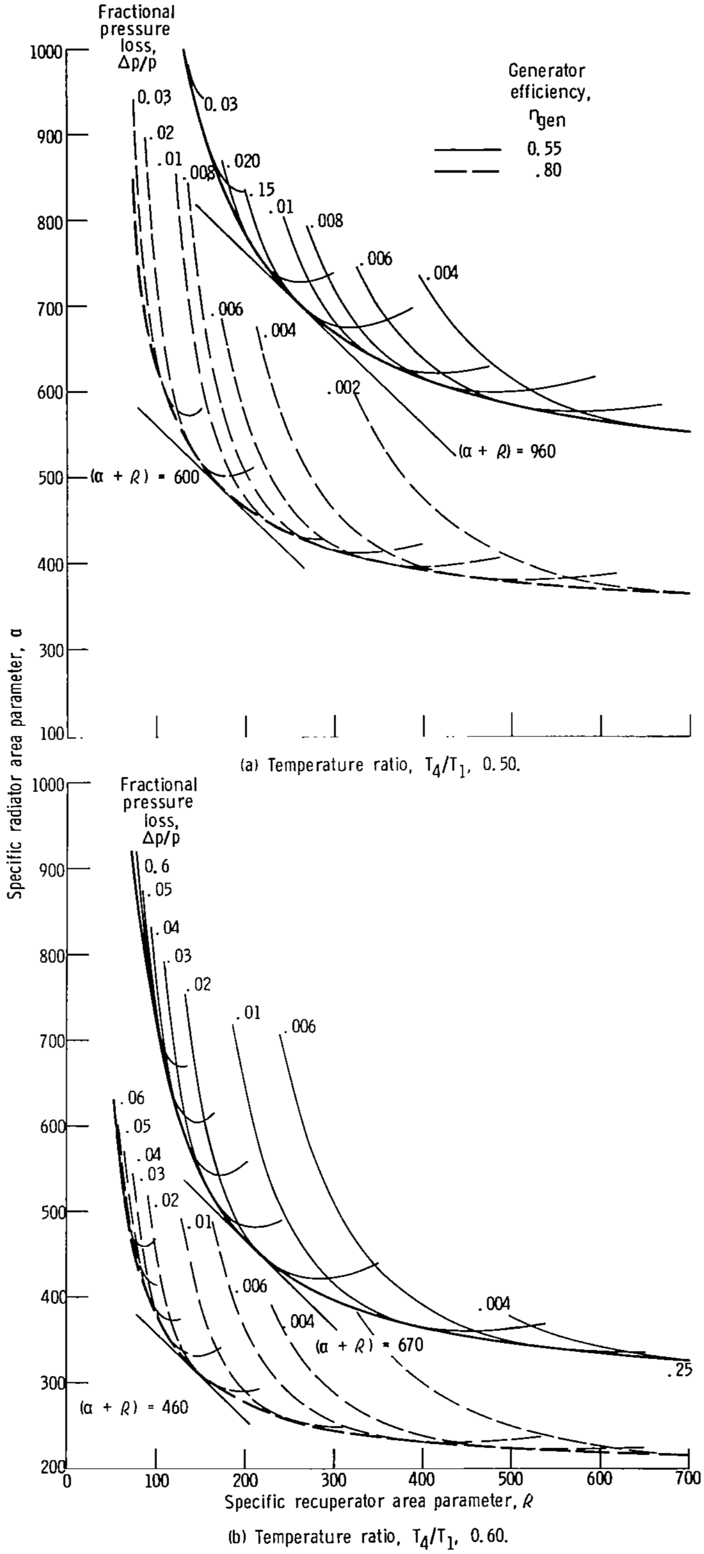


Figure 12. - Specific radiator area parameter plotted as function of specific recuperator area parameter for the combined cycle.

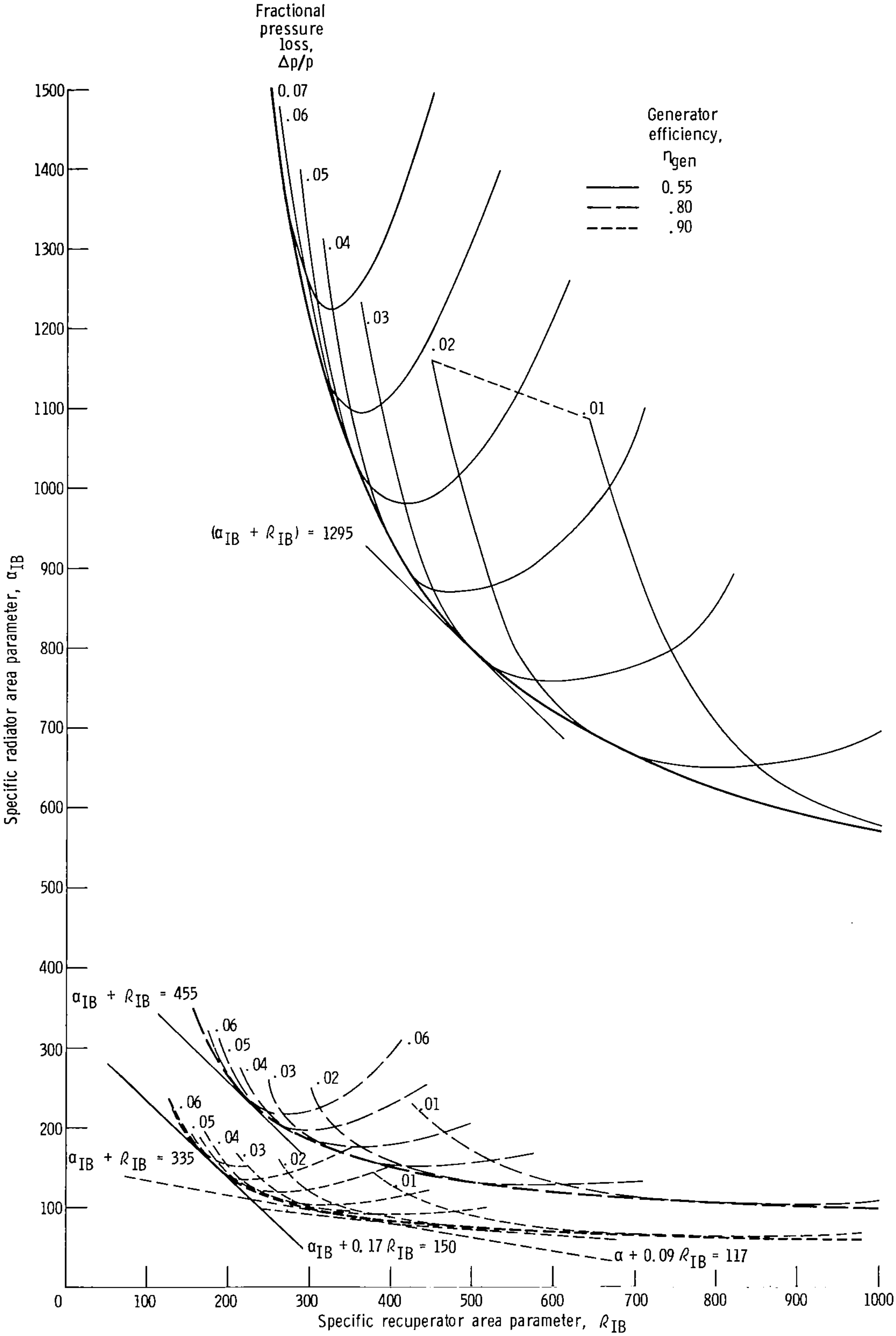


Figure 13. - Specific radiator area parameter as function of specific recuperator area parameter for intercooled Brayton cycle.

Comparison of Cycles for Space Use

The operating characteristics of eight cycles chosen for use in space can now be compared. The eight systems are divided among three types: all-MHD (system 1 and 2, combined turbo-MHD systems 3 to 6, and all-turbine systems 7 and 8. The first two groups have maximum temperature of 2500 K and two MHD efficiencies: 80 percent (systems 1, 3, and 5) and 50 percent (systems 2, 4, and 6). The combined systems consider two turbine-inlet temperatures: 1500 K (systems 3 and 4) and 1250 K (systems 5 and 6). The all-turbine systems assume a turbine efficiency of 90 percent and two turbine-inlet temperatures: 1500 K (system 7) and 1250 K (system 8). These conditions are summarized in table I. The system parameters which provide a minimum specific mass for the radiator and recuperator are given in table II.

If an m_{rad} of 8.4 kilograms per square meter (ref. 10) and an ϵ of 0.90 are chosen, then the total specific mass (kg/kW) can be calculated, and is plotted in figure 14.

The combined systems (3 to 6) have higher efficiency than the intercooled Brayton. This is the result of requiring that the turbine power be equal to the compressor power and is shown for a type of Carnot cycle in appendix C. This feature of the cycle may be important if the reactor mass can be reduced by reducing the power level. The combined cycles have lower specific recuperator and radiator mass - except for the high efficiency all-MHD generator system. However, the all-MHD system must also have an electric

TABLE I. - SPECIFICATION OF GENERATOR EFFICIENCY, GENERATOR INLET TEMPERATURE, AND TURBINE INLET TEMPERATURE FOR EIGHT SPACE POWER SYSTEMS

System description	System number	MHD generator inlet temperature, K	MHD generator efficiency, η_{gen}	Turbine inlet temperature, K
All MHD	1	2500	0.80	----
	2		.55	----
Combined	3	2500	0.80	1500
	4		.55	1500
	5		.80	1250
	6		.55	1250
All turbine	7	----	----	1500
	8	----	----	1250

TABLE II. - STATE POINTS AND OPERATING PARAMETERS FOR THE EIGHT SYSTEMS DEFINED IN TABLE I

[Compressor efficiency, 0.88; recuperator efficiency, 0.80; loss parameter, 0.95.]

System number	State point temperatures, K									
	Reactor outlet, T_1	Generator outlet, T_2	Reheater outlet, T_3	Turbine inlet[a], T_4	Turbine outlet[b], T_5	Low temperature recuperator inlet, T_8	Compressor inlet[c], T_9	Compressor outlet[d], T_{10}	High temperature recuperator inlet, T_{15}	Reactor inlet, T_{16}
1	2500	1619	----	----	----	1123	775	999	1494	----
2		1924	----	----	----	973	590	736	1687	----
3		1951	1561	1500	1130	857	542	789	1062	1120
4		2160	1808	1500	1166	852	550	773	1087	1380
5		2041	1769	1250	992	721	481	653	924	1417
6		2189	1922	1250	996	722	485	654	927	1565
7	1500	995	----	----	----	780	597	726	941	----
8	1250	835	----	----	----	667	521	627	793	----

System number	Generator efficiency, η_{gen}	Overall cycle pressure ratio, p_L/p_H	Fractional pressure loss, $\Delta p/p$	Dimensional parameter, r	Specific radiator area parameter, α	Specific recuperator area parameter, $\mathscr{R}$	Minimum specific mass parameter, $(\alpha + r\mathscr{R})_{min} = \mathscr{M}_{min}$	Total specific mass, m/P_e, kG/kW	Overall thermodynamic cycle efficiency, η
1	0.80	0.22	0.06	1	236	228	460	1.94	0.208
2	.55	.25	.03		805	497	1302	5.49	.169
3	.80	.085	.020		313	147	460	1.94	.398
4	.55	.110	.025		460	210	670	2.82	.304
5	.80	.135	.016		484	174	658	2.77	.424
6	.55	.140	.015		693	265	958	4.04	.332
7	----	.211	.033	.167	97	326	151	4.92	.211
8	----	.31	.021	.089	82	416	119	8.03	.212

[a] $T_4 = T_6$.
[b] $T_5 = T_7$.
[c] $T_9 = T_{11} = T_{13}$.
[d] $T_{10} = T_{12} = T_{14}$.

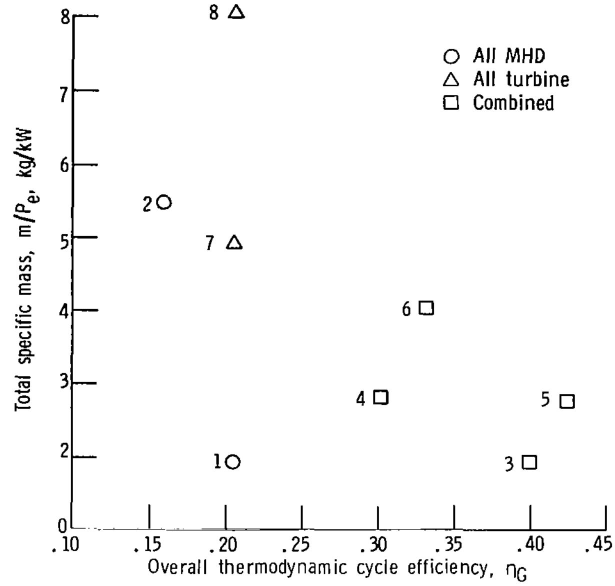


Figure 14. - Recuperator plus radiator specific mass as function of overall thermodynamic cycle efficiency. The numbers refer to systems whose characteristics are given in table I.

motor whose mass and inefficiency may result in a heavier system than the combined systems.

The radiator temperatures (T_8 and T_9) are lower for the combined system, for the same recuperator plus radiator specific mass (compare systems 1 and 3). This may result in a lower specific mass. The MHD generator outlet temperature is higher for the combined cycles, meaning that a higher power density can be achieved (ref. 6) and possibly a lighter magnet.

The overall cycle pressure ratio and fractional pressure drop are lower for the combined cycles. This may present a design problem for the heat exchangers. Comparison of the combined cycles indicates that reducing the turbine-inlet temperature from 1500 to 1250 K increases the mass the same as decreasing the generator efficiency from 80 to 55 percent. It is generally preferable to have the higher generator efficiency rather than the higher turbine inlet temperature, because the overall thermodynamic cycle efficiency is higher.

A decrease in generator efficiency from 0.80 to 0.55 increases the specific recuperator and radiator mass by a factor of 2.83 for the all-MHD intercooled Brayton cycle. A similar decrease in generator efficiency for the combined cycle resulted in a specific mass increase by only a factor of 1.45 for the combined cycle. The combined cycle specific mass is less sensitive to generator efficiency than the intercooled Brayton.

CONCLUDING REMARKS

There are several attractive features of the combined cycle compared with the intercooled Brayton cycle. Three of these are higher cycle efficiency, the relatively lower radiator temperature, and the lower sensitivity of the cycle performance to generator efficiency. The potential of these systems has been studied in more detail by Seikel and Nichols (ref. 11). They conclude that a 10-megawatt space system with specific mass of 3.5 to 5 kilograms per kilowatt electrical or ground-based systems with 60 percent efficiency could be achieved. However, a new technology would have to be developed. A 2500 K reactor is, of course, a necessity, but the NERVA program offers some encouragement in this area. A lightweight superconducting magnet is required, but recent work (ref. 11) indicates that advances in this area have been made. Finally, high efficiency MHD generators must be developed, but the chances of providing these generators is enhanced at the 2500 K temperature because of the increased conductivity of the gas.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, July 22, 1971,
129-02.

APPENDIX A

SYMBOLS

A	area
B	turbine parameter, defined in eq. (20b)
C	compressor parameter, defined in eq. (20c)
c_p	working fluid specific heat
E	expander parameter, defined in eq. (43)
f	friction factor
G	generator parameter, defined in eq. (20a)
h	heat transfer coefficient
L	heat exchanger loss parameter, defined in eq. (25)
$\mathscr{M}$	specific mass parameter of radiator plus recuperator
m	mass of radiator plus recuperator
N	number
P	power
p	working fluid pressure
Q	thermal power transferred to working fluid
R	gas constant for particular working fluid chosen
$\mathscr{R}$	recuperator specific area parameter
r	dimensionless parameter, defined in eq. (93)
S	entropy
s	fraction of entropy change, ΔS
T	working fluid temperature
U	overall heat transfer coefficient
u	working fluid velocity
w	working fluid mass flow rate
y	isentropic working fluid temperature ratio
α	specific radiator area parameter

γ	working fluid specific heat ratio
δ	pressure loss parameter, defined in eq. (4)
ϵ	radiator emittance
η	component efficiencies or effectiveness
ρ	working fluid density
σ	Stefan-Boltzmann constant

Subscripts:

a,b,a',b'	pressures in cycle
comp	compressor
cs	cross sectional area
e	electric
exp	expander
GB	ground based
gen	generator
H	high
h	high temperature recuperator
IB	intercooled Brayton
int	intercooler
L	low
min	minimum
Pr	Prandtl number
pri	primary
rad	radiator
rec	recuperator
turb	turbine
1, 2, . . . 15, 16	state points in cycle

APPENDIX B

COMPARISON WITH OTHER ANALYSES

Space Use

The results of the present study for the intercooled Brayton MHD cycle are compared with the results of Holman and Way (ref. 1) in table III. Holman and Way considered a cycle with no fractional pressure drop for the low-pressure working fluid in the piping and recuperator, and a value of 0.1 for the fractional pressure drop in the high-pressure fluid. If the same assumption is made for the present cycle, the efficiency obtained is higher and the specific radiator area parameter (and, hence, specific area since the temperature T_1 is the same for both systems) is slightly lower. The improvement is due to compressor intercooling. Increasing the compressor inlet temperature from 625 to 880 K lowers the efficiency and cuts the specific radiator area nearly

TABLE III. - COMPARISON OF CYCLE EFFICIENCY AND SPECIFIC RADIATOR AREA PARAMETER FOR INTERCOOLED BRAYTON CYCLE OF PRESENT STUDY WITH THE CYCLE CONSIDERED IN REFERENCE 3

[Compressor efficiency, 0.85; generator efficiency, 0.75; overall cycle pressure ratio, 0.444; recuperator effectiveness, 0.91.]

Source	Fractional pressure loss, $\Delta p/p$		Compressor inlet to reactor outlet temperature ratio, T_{13}/T_1	Overall thermodynamic efficiency, η	Specific radiator area parameter
	High-pressure working fluid	Low-pressure working fluid			
Brayton cycle (ref. 3)	0.1	0	0.25	0.3	370
Intercooled Brayton cycle of present study[a]	0.1	0	0.25 .35	0.36 .19	306 198
	0.1	0.1	0.25 .246	0.18 .19	739 747
	0.05	0.05	0.25 .31	0.31 .196	372 304

[a]The results of the present study are given for two values of T_{13}/T_1 for each set of $\Delta p/p$: one was chosen to equal the reference 3 value, the other set by minimizing the specific radiator area parameter.

in half. If a fractional pressure drop of 0.1 is assumed for both the high- and low-pressure lines, then the efficiency is about 0.18, and the specific radiator area is nearly twice as big as Holman and Way calculate. If, however, a fractional pressure drop of 0.05 in both the high- and low-pressure lines is assumed, the results for the compressor inlet temperature of 625 K are nearly identical with Holman and Way, and a slight decrease in specific radiator area can be achieved by increasing the compressor inlet temperature to 775 K (at the expense of a decrease in efficiency to about 0.20). This is illustrative of the sensitivity of the cycle results to the pressure drops allowed in the system. It may be that the present system is more sensitive than the Holman and Way system because of the intercooling in the compressor. A more detailed study would be required in order to assess the relative value of compressor intercooling.

The results are also compared with the turbine Brayton cycle study of Freedman (ref. 9). Freedman chose a generator efficiency of 0.8 and an equivalent turbine efficiency of 0.86. For a fractional pressure loss of 0.03 (B = 0.94 in Freedman's analysis), the minimum specific radiator for this cycle occurs at overall pressure ratio of about 0.4. At this condition the efficiency is 0.211, and for a turbine-inlet temperature of 1478 K the specific radiator area parameter is 148 for a specific radiator area of 6.1 ft^2/kW (0.56 m^2/kW). Freedman calculates an efficiency of about 0.196 at a specific radiator area of about 5 ft^2/kW (0.46 m^2/kW). The difference between the numbers is due to the compressor intercooling. From Freedman's analysis, it is possible to estimate the effect of temperature drops in the radiator. For a 285 K (50° F) outlet and 395 K (250° F) inlet temperature, the efficiency of Freedman's cycle drops to 0.181, and the specific radiator area increases to about 7 ft^2/kW (0.65 m^2/kW). The effect of these temperature drops in the cycle considered herein may be larger because of the intercooling. Again, more detailed analyses must be made in order to determine the exact effect.

Ground-Based Use

The results of the calculations for the ground-based system can be compared with the results of reference 2. Extrapolating the results presented in figure 4.12 of reference 2 (fig. 12 in the German reference) to a fractional pressure loss of zero, an overall cycle pressure ratio of 0.14 and a recuperator effectiveness of 0.78 provides a heat exchanger temperature difference of 150° K for the low-temperature recuperator and a resulting efficiency of 0.561. However, the assumption is made that no temperature difference occurs for the high-temperature recuperator. If the same assumption is made for the present system (i.e., $L_h = 1.0$), then the cycle efficiency of the present system is 0.572. It, therefore, seems fair to conclude that the two analyses are in agreement.

Bohn's analysis is very useful in showing the effect of the variation in many parameters for the ground-based system.

Bonsdorff et al. (ref. 4) calculate a net plant efficiency of 0.576 at a fractional pressure loss of 0.025. A recuperator effectiveness (for the low temperature heat exchanger) of 0.94 will provide the temperature difference of 40 K assumed by Bonsdorff et al. in their analysis. In order to relate their calculations to the present system, the diffuser efficiency (assumed by Bonsdorff et al. to be 0.75) must be included. Because of the small generator Mach number assumed by Bonsdorff, the correction will be small. The efficiency of 0.90 used by Bonsdorff for the generator must be changed to 0.875 in the present analysis to account for the diffuser pressure loss. The resulting cycle efficiency for the present cycle is 0.626.

Booth (ref. 9) assumed a heat exchanger temperature difference of 55.5 K. For the same conditions as given in figure 3 of Booth's paper, a recuperator effectiveness of 0.91 is required in the present analysis. The efficiency of the present system then works out to be 0.55, compared with Booth's 0.52. The difference might be in the temperature difference for the high-temperature heat exchanger. This difference is 334 K in Booth's analysis and 214 K in the present analysis.

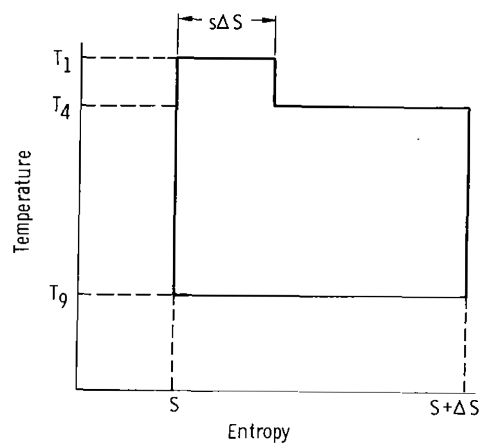
The Millionshchikov et al. (ref. 2) results are in general agreement with the present study. Not as much detail for the high-temperature system was presented, so that a detailed comparison cannot be made. The comparison that can be made (i.e., efficiencies in the range of 0.57 to 0.59) does seem to agree with the present results, however.

The comparison with other cycles, besides showing the good agreement with the results of the other studies, does provide some information about the values for certain parameters (such as the recuperator effectiveness, the fractional pressure loss, and compressor efficiency) that others seem to feel are typical.

APPENDIX C

COMPARISON WITH A TYPE OF CARNOT CYCLE

The efficiency that minimizes the specific radiator area of the combined system is higher than the efficiency of a Carnot cycle which minimizes the specific radiator area (25 percent). However, the combined system has two temperatures at which work is done on the working fluid. Thus, the Carnot cycle must be altered in order to make a comparison. The constant pressure heat exchange processes could be considered to be constant entropy processes if there were no frictional losses and if the heat were transferred with a recuperator effectiveness of 100 percent. The number of reheater and intercooling steps in the cycle could be increased, so that in the limit, at least, the expansion and compression processes could be considered to be isothermal. If the same limiting process were imagined for the MHD generator then an isothermal expansion could be assumed. These constant temperature processes would then occur at (from fig. 2) T_1 in the MHD generator, T_4 in the turbine, and T_9 in the compressor. A limiting cycle, which is Carnot-like, can then be conceived and is shown in the accompanying diagram



The efficiency of this cycle is

$$\eta = \frac{sT_1 + (1 - s)T_4 - T_9}{sT_1 + (1 - s)T_4} \tag{C1}$$

In order to compare this Carnot cycle with the combined cycle, it is important to require that the work delivered by the isothermal "turbine" stage, that is, $T_4(1 - s)\Delta S$, is made equal to the work required by the "compressor" stage, that is, $T_9 \Delta S$. Thus

$$T_9 = (1 - s)T_4 \tag{C2}$$

and the efficiency becomes

$$\eta = \frac{1}{1 + \frac{1 - s}{s} \frac{T_4}{T_1}} \tag{C3}$$

Notice that the minimum temperature T_9 appears only as it is determined by s from equation (C2). The radiator area and net electric power generated are related by

$$\epsilon \sigma T_9^4 A_{rad} = \frac{1 - \eta}{\eta} P_e \tag{C4}$$

The specific area parameter defined in equation (59), and the radiator temperature T_9 in equation (C2) can be introduced into (C4) and the efficiency η eliminated using (C3). Then, equation (C4) becomes

$$\alpha = \left(\frac{T_1}{T_4}\right)^3 \frac{1}{(1 - s)^3 s} \tag{C5}$$

In the combined cycle, the temperature ratio T_1/T_4 is held fixed. From equation (C5) it can be shown that α then achieves its minimum value at $s = 1/4$ and then that T_9 3/4 T_4. The minimum value of α is

$$\alpha_{min} = \left(\frac{T_1}{T_4}\right)^3 \frac{4^4}{3^3} \tag{C6}$$

and the efficiency is

$$\eta = \frac{1}{1 + 3 \frac{T_4}{T_1}} \tag{C7}$$

Notice that when $T_4 = T_1$, the values of α and η go to the usual Carnot result. The present type of Carnot cycle (with $T_4/T_1 < 1$) has greater minimum specific radiator area (at the same maximum temperature) than the usual Carnot cycle, but the corresponding efficiency is higher. This characteristic may prove beneficial for space application.

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